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ABSTRACT

The application of the concept of the representative strain is often used in the stress-strain curve determination from indentation test because it can significantly simplify the analysis of the indentation response. A new methodology for determining the representative strain for Vickers indentation is presented in this article. Following a procedure based on finite element simulations of indentation of elastoplastic materials, two representative strains are defined: the representative strain characteristic of the mean pressure and the representative strain characteristic of the Martens hardness or the indentation loading curvature. The results obtained from this methodology show that there is no universal value of representative strain independent of the mechanical parameters of materials indented by Vickers indentation. It is also shown that the representative strain, obtained by Vickers indentation is much lower when it is obtained from the relationship between the applied force and the penetration depth, *F-h*, rather than from the relationship between the applied force and the contact radius, *F-a*. The values of the calculated representative strains show that simultaneous measurement of relationships *F-a* and *F-h* make it possible to characterize the hardening law with two unknown parameters by Vickers indentation.

Key words: Mechanical properties determination; Vickers Indentation; representative strain; Constraint Factor; Hardness, Indentation curve.

1. Introduction

Indentation tests can be used not only for the evaluation of hardness, but also in the determination of other mechanical properties such as Young's modulus and stress–strain curves. The application of the concept of the representative strain can significantly simplify the analysis of the indentation response and was often used in the stress-strain curve determination from indentation test (Tabor, 1951; Giannakopoulos and Suresh, 1999; Venkatesh et al., 2000; Dao et al., 2001; Chollacoop et al.,2003; Bucaille et al., 2003; Kermouche et al., 2005; Ogasawara et al., 2005; Cao and Huber, 2006; Antunes et al., 2007; Kermouche et al., 2008). In the case of conical indentation, the representative strain, ε_R, is independent of the size of the indentation and depends on the half apex angle of the indenter, θ, which is equal to 70.3° for a conical indenter equivalent to the Vickers indenter. The studies performed on the representative strain in Vickers indentation can be divided into two groups, a first group which is based on the Mean Pressure (Tabor, 1951; Samuels and Mulhearn, 1957; Giannakopoulos et al., 1994; Chaudhri, 1998; Giannakopoulos and Suresh, 1999; Venkatesh et al., 2000; Mata et al., 2002; Kermouche et al., 2005; Kermouche et al., 2008; Branch et al., 2010) and a second group which is based on the Martens hardness (Dao et al., 2001; Bucaille et al., 2003; Chollacoop et al.,2003; Ogasawara et al., 2005; Cao and Huber, 2006; Antunes et al., 2007). The first group of studies concerns the definitions of the representative strain, which can lead to a relationship between a constant, C_F , called "the constraint factor", the Hardness, *H*, and the flow stress, σ_R, at a representative value of the plastic strain, ε_R, *i.e.* :

$$C_F = \frac{H}{\sigma_R} \tag{1}$$

In this relationship, *H* corresponds to the mean contact pressure, which is calculated from the diameter of the contact circle at full load (assumed to be equal to the diameter of the residual impression in the surface).

The second group of studies concerns the definitions of the representative strain, which can lead to a relationship between the reduced Young's modulus, E^*, the indentation loading curvature, C_L , and the representative stress, σ_R, *i.e.* (Dao et al., 2001):

$$C_L = \frac{F}{h^2} = \sigma_R \Pi_1 \left(\frac{E^*}{\sigma_R} \right) \tag{2}$$

In this relationship, the determination of the loading curvature, C_L leads to the determination of the "Martens" hardness, HM , which is equal to the following expression in the case of Vickers indentation:

$$HM = \frac{F}{26.43\, h^2} = \frac{C_L}{26.43} \tag{3}$$

The concept of representative strain was first introduced by Tabor (1951) to relate its corresponding representative stress to the Mean Pressure value. Tabor proposed, from experiments on essentially two materials, mild steel and copper, that the representative strain is equal to 0.08 in the case of Vickers indentation. This value is obtained so that the ratio of the Mean Pressure, *H*, to the corresponding representative stress, σ_R is equal to 3.3 (value previously determined from experiments performed on work-hardened metals) (Tabor, 1951). The value of 0.08 proposed by Tabor is similar to the value of 0.07 reported by Samuels and Mulhearn (1957) in the case of Vickers indentations in annealed 70:30 brass. This proposition is also close of the numerical results obtained by Mata et al. (2002) for conical indentation of elastic plastic materials with various Young modulus, *E*, Yield stress, σ_y, and hardening exponent *n*, i.e. ε_R=0.1. With this value of representative strain, the ratio of the hardness, *H* to the corresponding representative stress, σ_R, was found equal to 2.7. For Mata et al. (2002), the accuracy of Tabor's equation is limited to the fully plastic contact regime. As long as this regime prevails, Tabor's equation, i.e. $H/\sigma_{(\varepsilon=0.08)}$=3.3, is found to be extremely accurate as hardness values estimated by this equation.

From an experimental investigation of the surface and subsurface strain hardening around Vickers indentations in annealed copper, it was determined that the maximum plastic strain occurs in a subsurface region close to the indentation tip where the estimated plastic natural strain is in the range from 0.25 to 0.36 (Chaudhri, 1998). Srikant et al. (2006) found similar values of maximum strain for similar experimental conditions (maximum plastic strain in the range between 0.22 and 0.31). Chaudhri (1998) suggest that the equivalent strain associated with a relatively large Vickers indentation should be 0.25-0.36 for annealed metals having a power law uniaxial stress vs strain relationship. Moreover, finite element computations using a conical indenter equivalent to the Vickers indenter (θ=70.3°) show that the equivalent plastic strain within a 7075-T651 aluminium exceed 15% in the majority of the volume directly beneath the indenter. Giannakopoulos et al. (1994), Giannakopoulos and Suresh (1999) and Venkatesh et al. (2000) used a “characteristic strain” of 29–30% within the context of their formulation. Giannakopoulos and Suresh (1999) suggested that the region of material experiencing strains beyond 29% under the indenter exhibits plastic “cutting” characteristics and may be modelled using slip line theory. These values are considerably higher than the value of 0.08 proposed by Tabor (1951) almost 60 years ago. Tabor's proposal was based on a fundamental assumption according to which the ratio of Vickers hardness to uniaxial Flow stress, corresponding to any prior strain plus an additional strain introduced by the indentation process, should be universally constant and equal to 3.3. This original definition does not represent any apparent physical transition in mechanical response. Moreover, this assumption has not been fully justified so far, experimentally or theoretically. Chaudhri (1998) also shows that there is very little difference between choosing ε_r =0.08 and ε_r = 0.2 as far as the ratio of the Vickers hardness to the flow stress is concerned. For Chaudhri (1998), ε_R =0.08 is not a unique value of the equivalent strain introduced by a Vickers indentation. He suggests that a better choice of the equivalent strain should be related to the maximum strain produced in the deformed zone. For Branch et al. (2010), the best choice is rather the volume average plastic strain within the plastic zone of Vickers indentation. Some authors have concluded that the Mean Pressure does not depend on a unique representative strain. Dugdale (1958), who investigated the stress-strain curves and Vickers hardness of a number of metals, alloys and nylon, has proposed that the stress-strain curves up to a strain of 0.15, and not just the stress corresponding to a single value of strain, are relevant in predicting their Vickers hardness values. For Larsson (2001), at indentation of rigid-plastic power-law materials, the hardness is well-described by a single representative strain level in the spirit of Tabor. In this case, the Vickers Hardness calculated with the constant values C_F= 2.55 and ε_R=0.18 or C_F= 2.8 and ε_R=0.15 are in fairly good agreement with the numerical results. In a general situation, i.e. at indentation of materials with more irregular stress-strain relations, Larsson (2001) found that the concept of a single representative strain is no longer valid. For this general situation, an alternative two-parameter description of the Mean Pressure is suggested with the two parameters corresponding to the stress levels at 2 and 35% plastic strain. To conclude, the different studies on a material-dependent representative plastic strain valid in the conversion of flow stress to Mean Pressure suggest that there may not be a universal value for the equivalent strain introduced by a Vickers indentation.

In the case of “Martens” hardness, Dao et al. (2001) shows that the value of the “representative” strain depends on the choice of functional definitions that is used to relate certain indentation parameters to certain mechanical properties. Using dimensional analysis, a set of new universal dimensionless functions was constructed to characterize instrumented sharp indentation. Based on this dimensional analysis, a representative plastic strain ε_R = 0.033 was identified as a strain level which allows for the construction of a dimensionless description of the indentation loading response, independent of strain hardening exponent n (Eq. (2)). In their work, Dao et al (2001) discuss about the underlying connections between the different functional definitions given for conical indentation (Tabor, 1951; Giannakopoulos et al., 1994; Giannakopoulos and Suresh, 1999; Dao et al., 2001) and the corresponding representative strain levels. It was shown that the differences in the magnitude of the strain came from their different functional definitions. For example, the work performed by Dao et al. (2001) demonstrates clearly that the representative strain is not the same if we consider the “Martens” hardness (3.3%) or the Mean Pressure (8%).

Cao and Huber (2006) show that different definitions of the representative strain, can lead to a one-to-one relationship with high level of accuracy between the reduced Young’s modulus, the indentation loading curvature and the representative stress.

For methods using the energy-based representative strain and methods using a stress-state-based definition of the representative strain, Cao and Huber (2006) reported ε_r values in the range 0.023–0.095. Lastly, Antunes et al. (2007) reported material-dependent representative plastic strain values ranging between 0.034 and 0.042.
Clearly, the values for representative plastic strain vary over a broad range. These values depend on the choice of the functional parameters that were used to describe the indentation process (Martens hardness or mean pressure) and were obtained from curve fitting the indentation responses of a certain range of material properties. It seems that, as for the Mean Pressure, the “Martens” hardness can not be determined from Eqs. (2) and (3) if a universal value for the equivalent strain introduced by a Vickers indentation is used.

The present work is conducted with the objective of studying the concept of universal value for the representative strain introduced by a conical indentation equivalent to the Vickers indentation. A finite element study on elasto-plastic materials with a Hollomon hardening behaviour is presented in order to define news values of representative strain. Two representative strains will be define: the representative strain characteristic of the mean pressure and the representative strain characteristic of the Martens hardness or the indentation loading curvature.

2. Numerical method and new definition of the representative deformation

2.1. F.E. model

The finite element analysis presented here assumes a conical perfectly rigid indenter in frictionless contact with the flat surface of the specimen (Fig. 1). The simulations were performed in axisymmetric mode using the large strain elastic-plastic feature of the Abaqus finite element code.
The included half apex angle, θ, of the rigid conical indenter was 70.3° (see Fig. 1). This value of apex angle gives an identical relationship of contact area–depth of penetration as the Vickers indenter.
12516 four-noded axisymmetric quadrilateral elements made up the FE models, with the finest mesh in the region of the indented material. There were 300 elements in contact with the indenter during maximum indent depth, which provided sufficient resolution (Fig. 1). The maximum depth of penetration was chosen so that, in all cases, the contact radius be 4920 times smaller than the total length of the mesh.
The constitutive model of the elastic-plastic indented material was taken to follow the well known J_2-associated flow theory with rate-independent deformation. The isotropic hardening is described by the Hollomon power law, expressed by Eq. (4).

$$\begin{aligned} &\sigma = E\varepsilon && (Hooke) && if \quad \varepsilon < \sigma_y / E \\ &\sigma = E^n \sigma_y^{1-n} \varepsilon^n && (Hollomon) && if \quad \varepsilon \geq \sigma_y / E \end{aligned} \tag{4}$$

where E is the Young modulus, σ_y the yield stress and n the hardening coefficient.

The finite element simulations were performed for materials with Poisson’s values v= 0.3.
In the simulation, 272 different combinations of elastic plastic properties listed in Table 1 were investigated to determine the Mean Pressure and the Martens hardness and corresponding representative strains.

E (MPa)					
210000					
σ_y (MPa)					
10	16	25	40	63	100
160	250	400	630	1000	1600
2500	4000	6300	10000		
n					
0	0.0125	0.025	0.05	0.075	0.1
0.125	0.15	0.2	0.25	0.3	0.35
0.4	0.45	0.5	0.55	0.6	

Table 1: Material properties used in the present computations, Poisson ratio v =0.3.

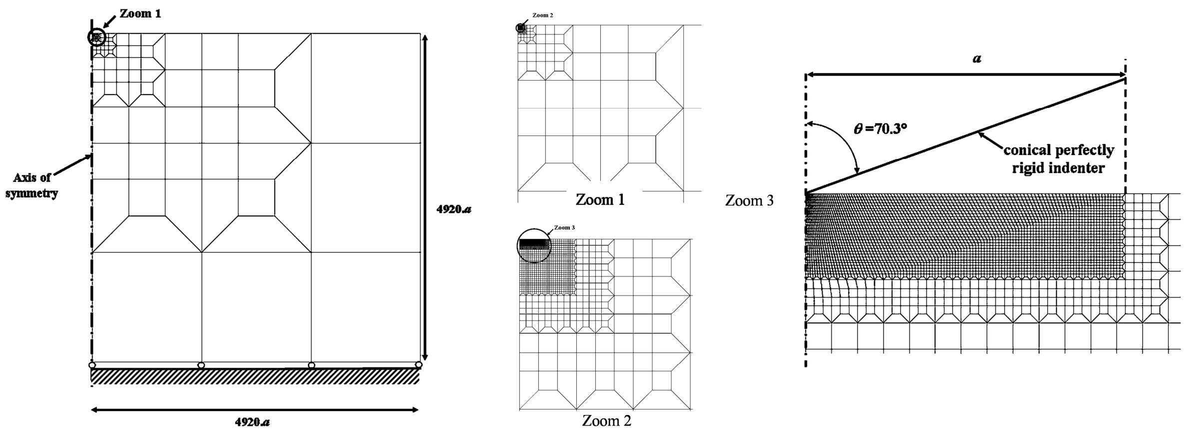


Fig. 1 : Typical finite-element mesh, composed of four-noded axisymmetric elements and the rigid indenter with an equivalent half cone angle of 70.3°: overall mesh and detail in the region of contact.

2.2. Relationships between representative strain and Mean Pressure or "Martens" hardness

In the various studies on the stress-strain curve determination from indentation test, the representative stress was calculated from a plastic equivalent strain, ε_{RP} (Tabor, 1951; Giannakopoulos et al., 1994; Chaudhri, 1998; Giannakopoulos and Suresh, 1999; Venkatesh et al., 2000; Larsson, 2001; Cao and Huber, 2006; Branch, 2010), from a total equivalent strain, ε_R (Mata et al., 2002), or from a "characteristic" strain corresponding to the nonlinear part of the total effective strain accumulated beyond the yield strain, ε_{R0} (Dao et al., 2001; Cao and Huber, 2006).

In this work, it is considered in a first step that the representative strain, ε_R, corresponds to the total strain "characteristic" of the Vickers indentation.

2.2.1 Mean Pressure

Fig. 2 shows the values of the H/E dimensionless Mean Pressure obtained for the studied materials.

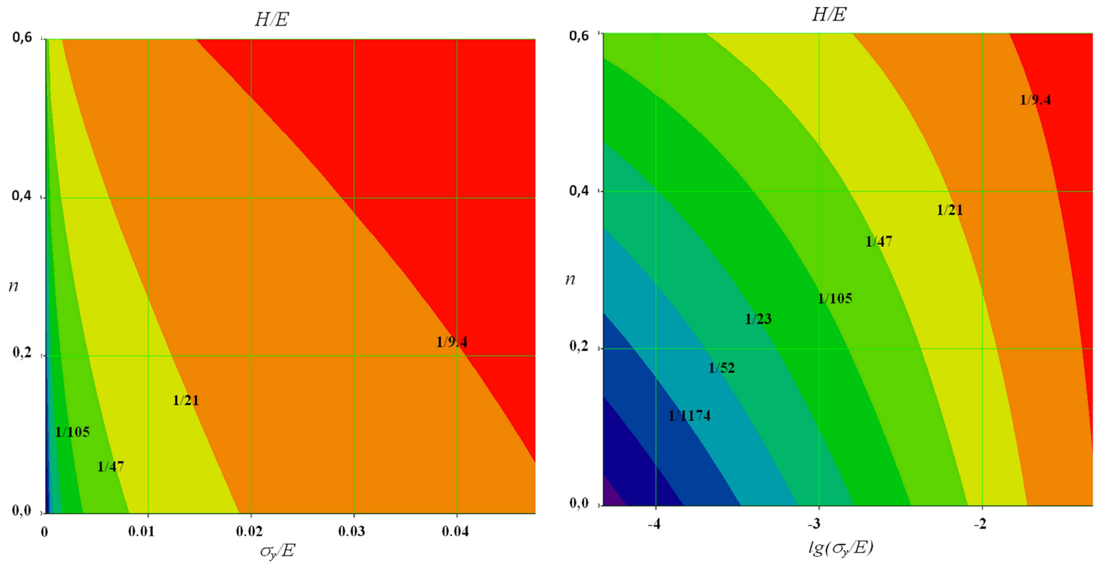


Fig. 2 : Values of the H/E dimensionless Mean Pressure in function of σ_y/E (in linear and logarithmic scale) and n.

Fig. 2 shows that the increase in Mean Pressure with σ_y/E (in linear and logarithmic scale) and n is non linear.

When the Yield stress is exceeded, Eqs. (1) and (4) give:

$$\ln\left(\frac{H}{E}\right) = n\ln(\varepsilon_R) + (1-n)\ln\left(\frac{\sigma_y}{E}\right) + \ln(C_F) \tag{5}$$

Substituting $K = (1-n)\ln\left(\frac{\sigma_Y}{E}\right)$ in Eq. (5) leads to:

$$\ln\left(\frac{H}{C_F\,E}\right) = \ln(\varepsilon_R)\,n + K \tag{6}$$

With the assumption that the representative deformation, ε_R, is constant, Eq. (6) shows that a linear relationship is obtained between n and K for materials of same $H/(C_F E)$ ratio. Fig. 3 represents the values of $\ln(H/E)$ obtained by F.E.M. in the n-K diagram. In this figure, it is seen that $\ln(H/E)$ is about constant following a straight line in the n-k diagram. The deviation from the constancy of $\ln(H/E)$ following a straight line in the (n-K) 2 dimensions space is due to the variations of the representative strain, ε_R and constraint factor, C_F.

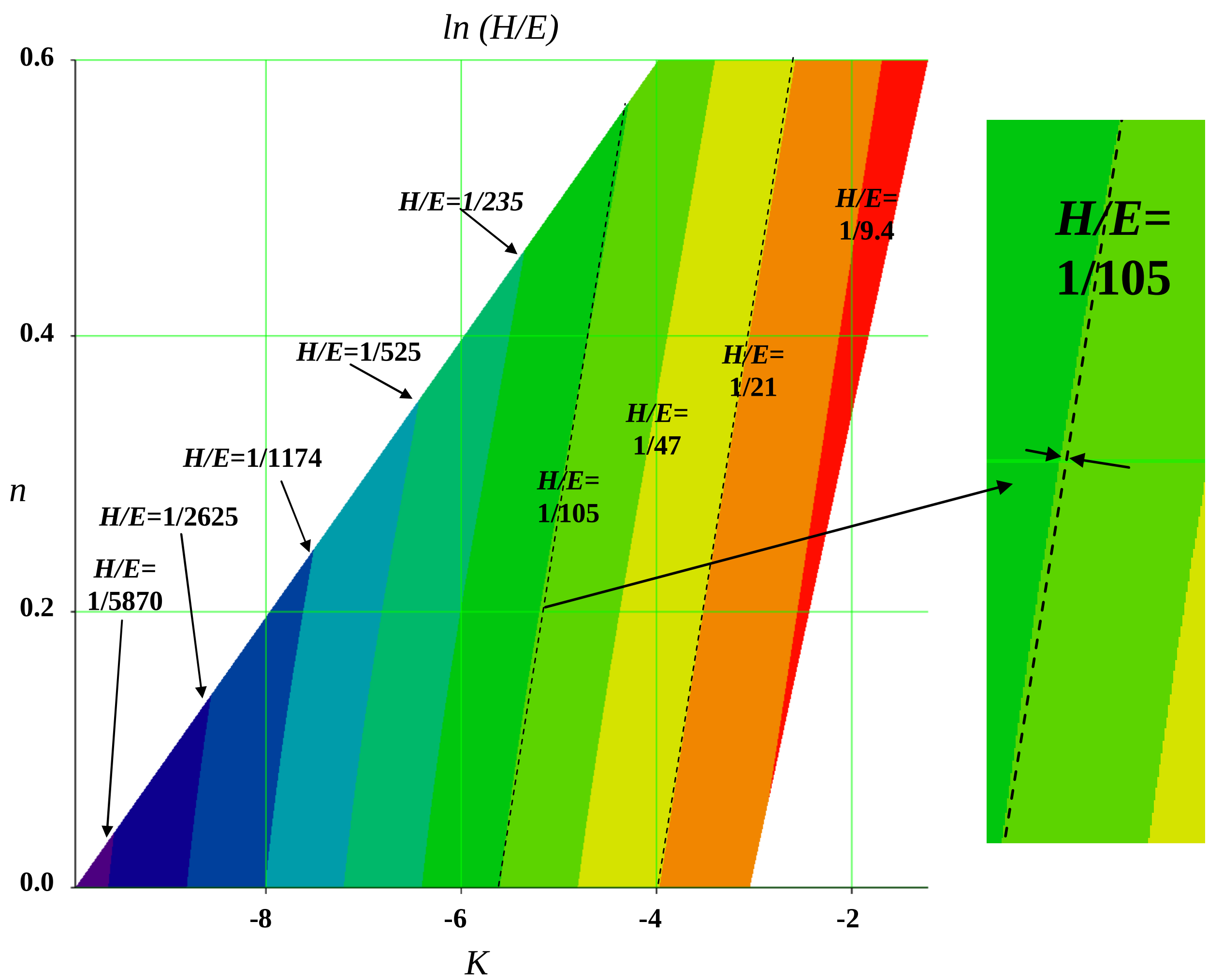


Fig. 3: H/E ratio values obtained for the various values of *n* and *K* parameters studied in this work.

Fig. 3 shows that the constancy of $\ln(H/E)$ following a straight line in the (n-K) 2 dimensions space is obtained for materials of H/E ratio equal to $1/21$. This result indicates that the representative strain, ε_R, and the constraint factor, C_F, are almost constants for these materials. Fig. 4 confirms that the dimensionless stress-strain curves $(\sigma/E-\varepsilon)$ of these materials intersect approximately at a same value of strain, ε_R. Constant σ_R/E and constant H/E ratio has for consequence the constancy of C_F for materials of H/E ratio equal to $1/21$.

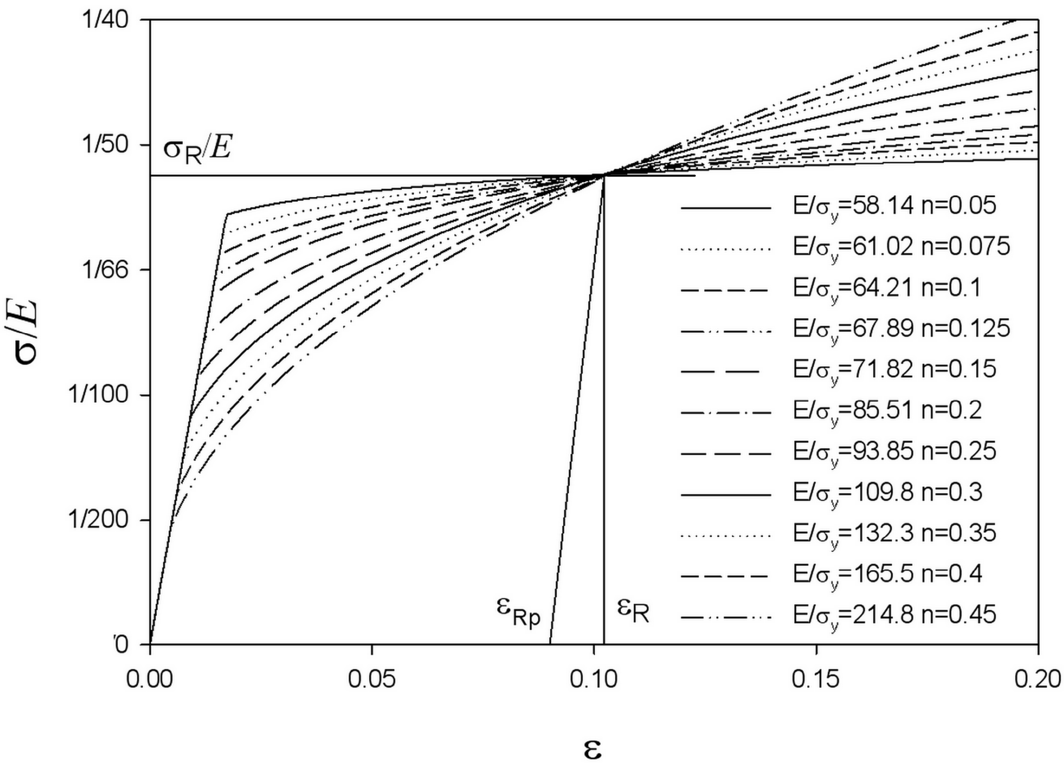


Fig. 4: $\sigma/E-\varepsilon$ curves for materials with $H/E=1/21$ in Vickers indentation.

On the other hand, Fig. 3 shows that the constancy of $\ln(H/E)$ following a straight line in the (n-K) 2 dimensions space is not obtained for materials of H/E ratio equal to $1/105$. This result indicates that the representative strain, ε_R, and the constraint factor, C_F, are not constants for these materials. Fig. 5 confirms that the dimensionless stress-strain curves $(\sigma/E-\varepsilon)$ of these materials are not intersect at a same value of strain, ε_R. Non constant σ_R/E ratio and constant H/E ratio has for consequence the non constancy of C_F for materials of H/E ratio equal to $1/105$.

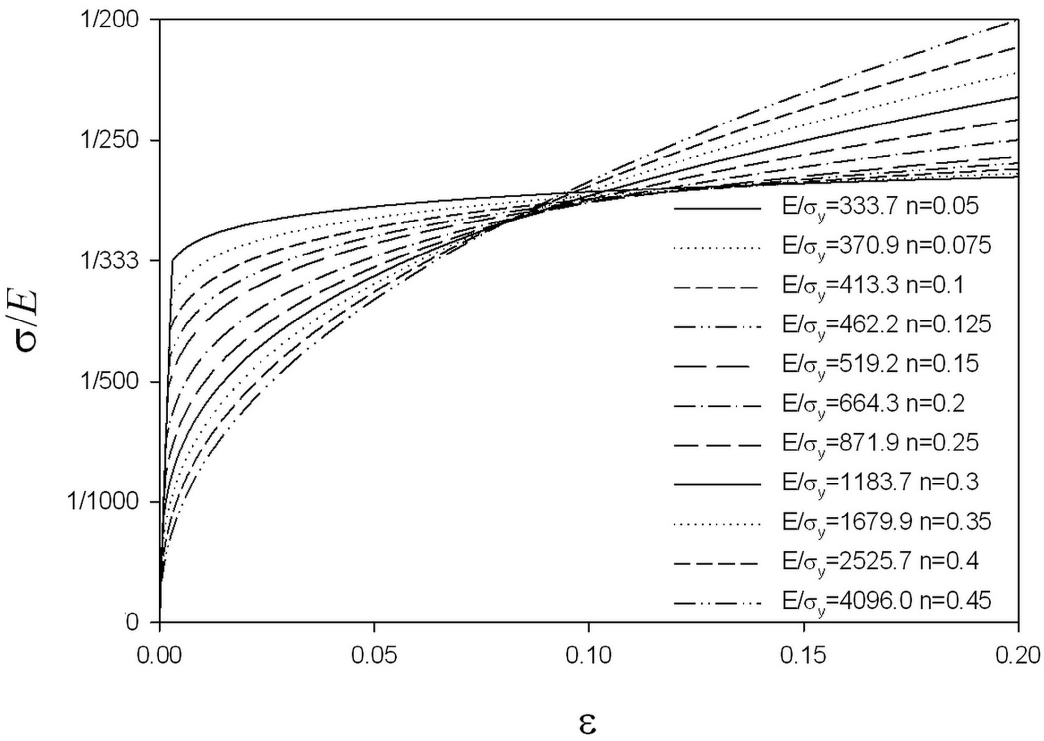


Fig. 5: $\sigma/E-\varepsilon$ curves for materials with $H/E=1/105$ in Vickers indentation.

If it is considered that the constraint factor, C_F, is locally constant for materials of same H/E ratio, the representative strain can be determined for materials of same H/E ratio by differentiation of Eq. (6). This differentiation gives:

$$\varepsilon_R=\exp\left(-\left(\frac{dK}{dn}\right)_{H/E}\right) \tag{7}$$

2.2.2. "Martens" hardness

It can be deduced from Eq. (2) that the representative stress is:

$$\sigma_R=C_L\Pi_2\left(\frac{C_L}{E^*}\right) \tag{8}$$

Following the same procedure as for the Mean Pressure, Eqs. (3), (4) and (8) give:

$$\ln\left(\left(\frac{HM}{E}\right)\Pi_3\left(\frac{HM}{E^*}\right)\right)=\ln(\varepsilon_R)n+K \tag{9}$$

In the case of conical rigid indenter, Eq. (9) shows that the representative strain can be determined for materials of same HM/E ratio and same Poisson's ratio by using Eq. (7).

Fig. 6 represents the values of $\ln(HM/E)$ obtained par F.E.M. in the (n-K) diagram.

This figure is similar to Fig. 3, *i.e.*, $\ln(HM/E)$ is about constant following a straight line in the (n-K) diagram. The deviation from the linear variation of $\ln(HM/E)$ in the (n-K) 2 dimensions space is due to the variations of the representative strain, ε_R .

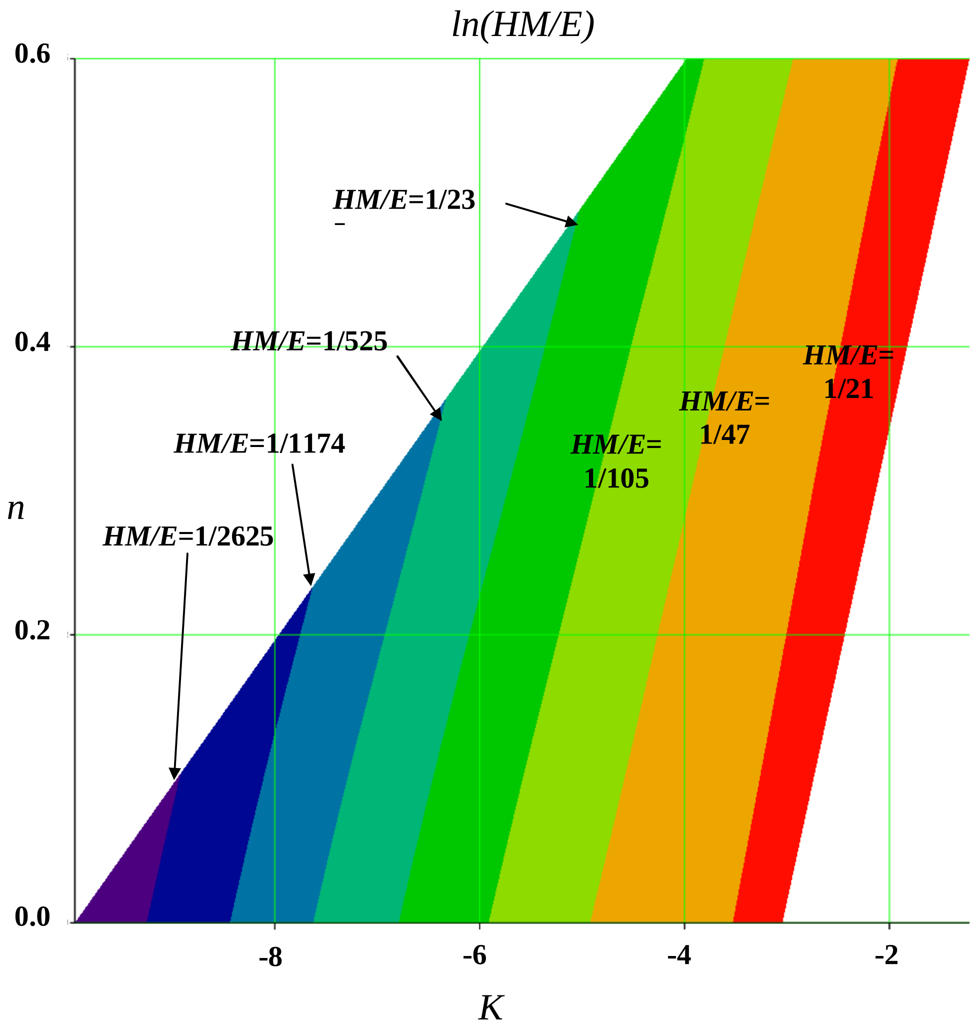


Fig. 6: HM/E ratio values obtained for the various values of *n* and *K* parameters studied in this work.

2.3 Representative strain determination from F.E.M. results

The values of representative strain were obtained from Eq. (7) and from the numerical results of Mean Pressure, *H*, and "Martens" hardness, *HM* by using the following procedure.

Figs. 3 and 6 show that the natural logarithm of Mean Pressure, *H*, and "Martens" hardness follow locally a straight line in the (n-K) diagram. It can be deduced from these figures that, locally:

$$\ln\left(\frac{H \text{ or } HM}{E}\right) = \alpha n + \beta K + \chi \tag{10}$$

In order to determine the values of α, β and χ of this equation, the (n-K) diagram was divided in small trapezoids with vertices defined by the four closest numerical values of (n-K). For each trapezoid, the values of α, β and χ are obtained by the minimisation of the following system of equations with the least square method:

$$\sum_{i=1}^{4}\left(\ln\left(\frac{H_i\ or\ HM_i}{E}\right)-\alpha n_i-\beta K_i-\chi\right)^2 \tag{11}$$

Where H_i and HM_i are respectively the Mean Pressure and "Martens" hardness at the four vertices of each trapezoid. With the assumption that the constraint factor, C_F, and that the dimensionless parameter, $\Pi_3\left(HM/E^*\right)$, are constant in each trapezoid respectively for materials of same H/E ratio and same HM/E, the representative strain can be deduced for each trapezoid from Eq. (7) with:

$$\frac{\partial\left(\ln\left(\left(H_i\ or\ HM_i\right)/E\right)\right)}{\partial n}=\alpha \quad ; \quad \frac{\partial\left(\ln\left(\left(H_i\ or\ HM_i\right)/E\right)\right)}{\partial K}=\beta \tag{12}$$

3. Results and discussion

3.1 Mean Pressure

3.1.1 Representative strain

Fig. 7 shows the values of the representative strain calculated from the Mean Pressure by using Eqs. (7) and (12).

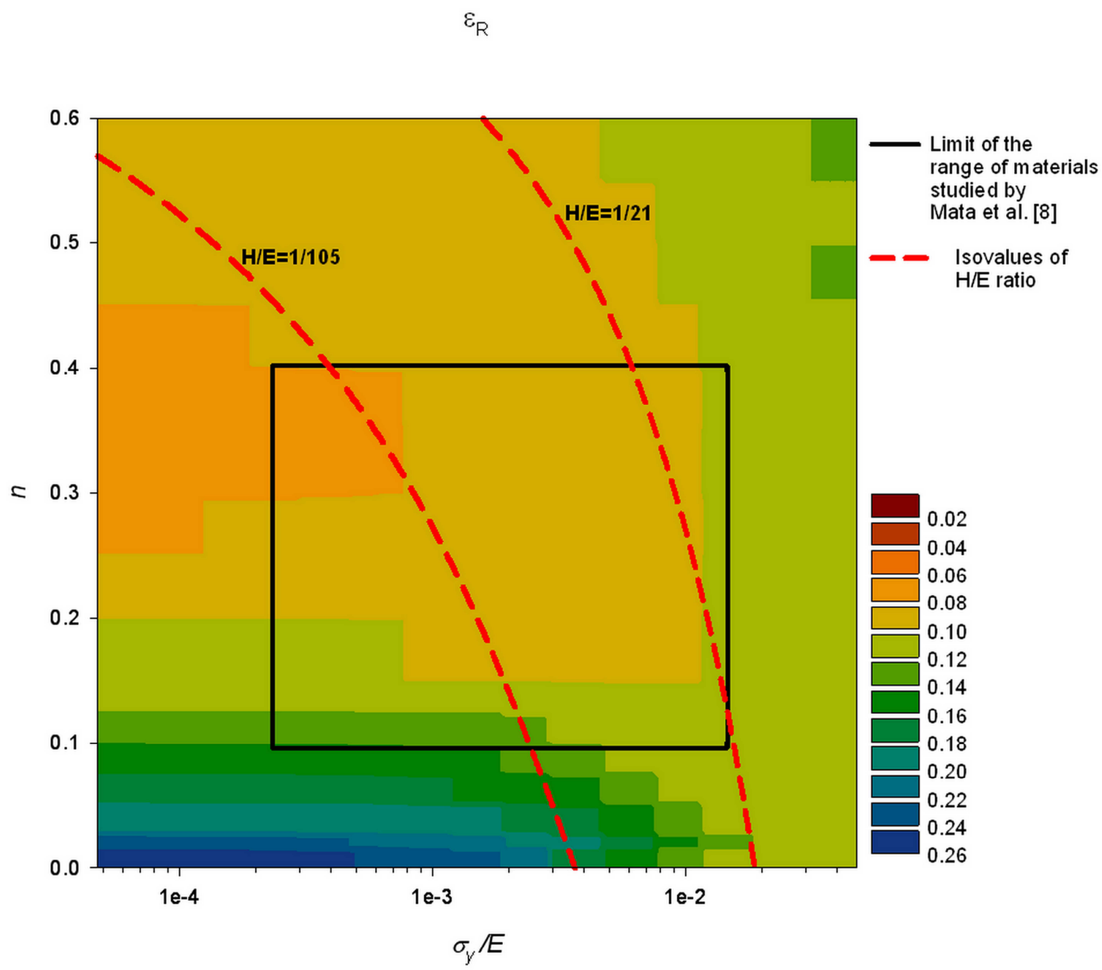


Fig. 7: Representative strain calculated from the Mean Pressure (total equivalent strain).

From Fig. 7, it is clearly shown that the universal value for the equivalent strain introduced by a Vickers indentation doesn't exist if we consider the Mean Pressure. Fig. 7 confirms that the representative strain is about constant for material with $H/E = 1/21$ and $0. \leq n \leq 0.45$. This figure also confirms that the representative strain vary depending on the hardening exponent for material of H/E ratio equal to $1/105$. Moreover, Fig. 7 shows that the representative strain is not constant for the materials studied by Mata et al (2002). ε_R=0.1 proposed by Mata et al corresponds to an average value of the representative strain obtained for the different materials studied by these authors. More generally, the representative strain is in the range from 0.2 to 0.25 for materials of small hardening exponent and small σ_y / E ratio. For materials of large hardening exponent and large σ_y / E ratio the representative strain is about equal to 0.1.
In order to compare with the Tabor's value (1951), i.e. 0.08, the representative plastic strain obtained with our procedure is shown Fig. 8.

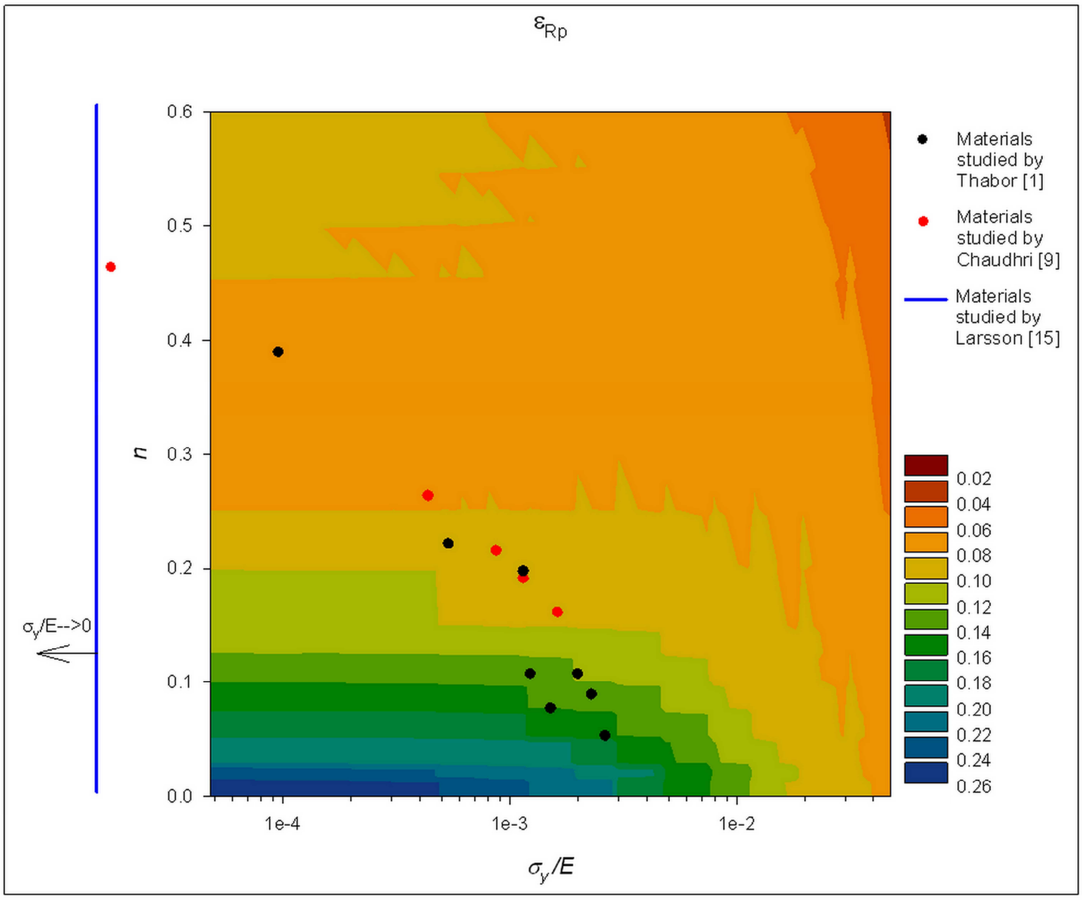


Fig. 8: Representative plastic strain calculated from the Mean Pressure.

Fig. 8 shows that ε_{RP}=0.08 is in fairly good agreement with the numerical results for materials with large work-hardening exponent. The comparison between our results and those obtained by Chaudhri (1998) also shows that the representative strain value which corresponds to the maximum plastic strain in the indented plastic zone, i.e. plastic natural strain in the range from 0.25 to 0.36, is very higher that the represented plastic strain value obtained from our procedure. This indicates that the choice of the maximum plastic strain as a representative

deformation is not correct in order to determine the stress-strain curve of materials from the Mean Pressure measurement. Concerning the representative plastic strain of Larsson (2001), i.e. ε_{RP}=0.15-0.18 , this one corresponds to the average of those obtained from our procedure which are in the range between 0.09 and 0.26 (see Fig. 8).
For materials with low work-hardening exponent, the represented plastic strain values obtained from our procedure are higher than the Tabor's value, i.e. ε_{RP}=0.08. It is useful to remind that Tabor's proposal was based on a fundamental assumption according to which the ratio of mean contact pressure to uniaxial flow stress, corresponding to any prior strain plus an additional strain introduced by the indentation process, should be universally constant and equal to 3.3. For material of low work hardening exponent, there is very little difference between choosing ε_{RP}=0.08 and the representative strain obtained from our procedure as far as the ratio of the mean contact pressure to the flow stress is concerned.

3.1.2. Constraint Factor

The values of constraint factor obtained from Eq. (1) and the values of representative strain shown Fig. 7 are given in Fig. 9.

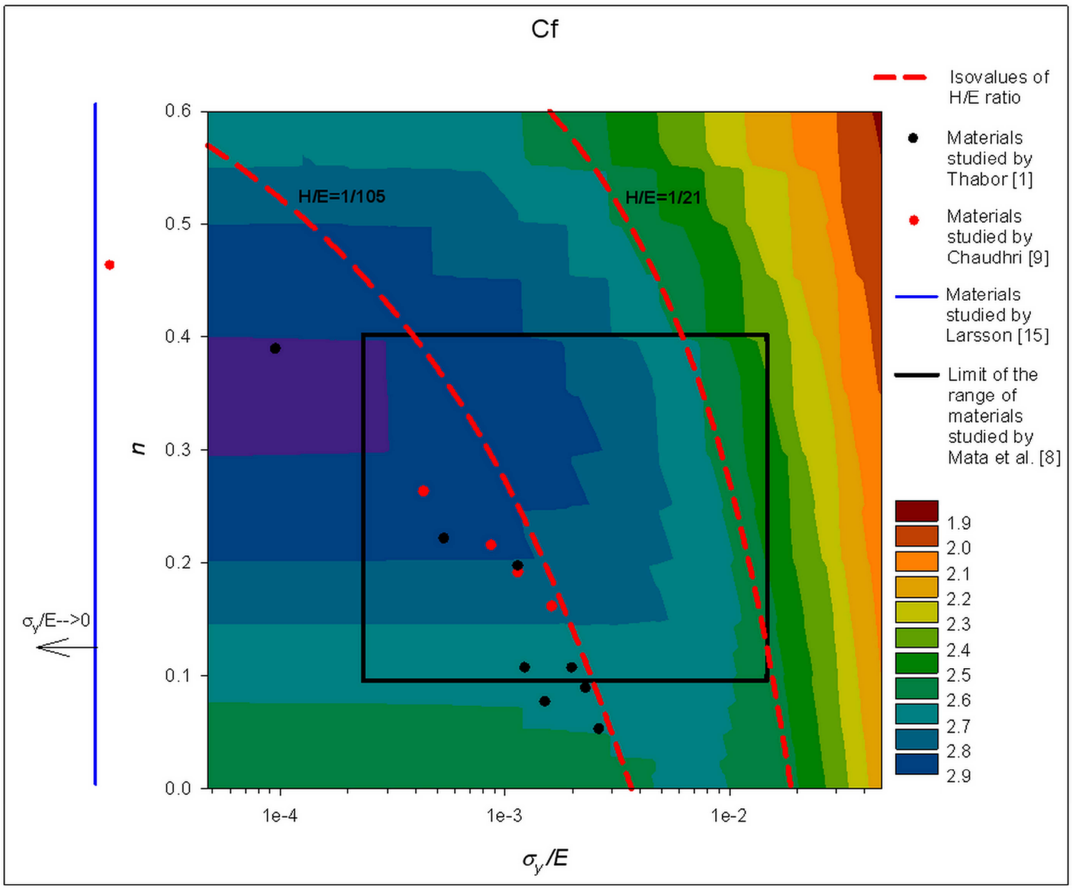


Fig. 9: Values of the Constraint Factor, C_F , obtained from Eq. (1) and the values of representative strain.

As for the case of the representative strain, Fig. 9 confirms that the constraint factor is about constant for material with $H/E = 1/21$ and $0. \le n \le 0.45$. This figure also confirms that the constraint factor vary depending on the hardening exponent for material of H/E ratio equal to $1/105$. From Fig. 9, it is clearly shown that the universal value for the constraint factor introduced by a Vickers indentation doesn't exist if we consider the Mean Pressure. $C_f = 2.7$ was obtained by Mata et *al.* (2002) under frictionless condition and with the assumption that $\varepsilon_R = 0.1$. Fig. 9 shows that this value is close to the different values of constraint factor obtained from our procedure for the materials studied by Mata et al. (2002). Fig. 9 also shows that C_F= 2.8 obtained by Larsson (2001) for indentation of rigid-plastic power-law materials under frictionless condition corresponds to the average of those obtained from our procedure which are in the range between 2.6 and 2.9. By using the Tabor approximation (i.e. $\varepsilon_R = 0.08$), the ratio of the Mean Pressure to flow stress, σ_R, was found to be about equal to 3.3 in the case of Vickers indentation of work-hardened steel, cooper or aluminium (Tabor, 1951). In the case of work-hardened Copper, Chaudhri (1998) found that the ratio of the Mean Pressure to σ_R is approximately constant at 3.0-3.5 depending on the value of the representative strain. Fig. 9 shows that the numerical values of C_F determined from our procedure are lower than the experimental values of C_F proposed by Tabor and Chaudhri. Compared to our results, the larger values of C_F found by Tabor and Chaudhri from experimental test can be due to the effect of the friction coefficient, μ, on the indentation response of the studied materials. The effect of friction on the indentation response will be studied in the following section.

3.1.3 Effect of friction on the representative strain and constraint factor values

From numerical and experimental investigations of conical indentation with half angle greater than 60 °, it was concluded that adhesion and friction between the indenter and the substrate were found to have only a small effect on the hardness and the *P-h* curve (Li et al., 1993; Giannakopoulos et al., 1994; Larsson et al., 1996; Giannakopoulos and Larsson, 1997; Mata and Alcala, 2004; Antunes et al., 2006). Only a small increase in hardness occurs for friction contacts as compared to frictionless ones (Li et al., 1993; Mata and Alcala, 2004; Antunes et al., 2006). For example, Antunes et al. (2006) found no variation of hardness between three-dimensional numerical simulations of Vickers indentation performed with $\mu = 0.08$ and $\mu = 0.24$ and Giannakopoulos et al. (1994) and Mata et al (2002) found that friction increases by less than 8% the average contact pressure obtained for frictionless indentation. The small effect of the friction on the hardness can be explain by the fact that the tangential displacements at the contact region were very small compared to the vertical ones, except very close to the tip of the indenter (Giannakopoulos et al., 1994; Larsson et al., 1996). Despite that only a small effect of friction on the hardness was observed, friction between indenter and half space reduces the amount of pile-up at the edge of the indenter and influences considerably the stress and strain fields in the proximity of the contact region (Mata and Alcala, 2004; Antunes et al., 2006). From three-dimensional numerical simulations of Vickers indentation of AISI steel and Nickel, Antunes et al. found that the distribution of the equivalent plastic strain under the indenter is quite dependent on the value of the friction coefficient. For low values of the friction coefficient ($\mu = 0.04$), the maximum value of the equivalent plastic strain is quite high (≈ 1.27), and it is located on a small area at the surface of the indentation. In the case of a high friction coefficient ($\mu = 0.24$), the maximum value of

the equivalent plastic strain (≈ 0.38) is lower and it is located not only at the surface but also at a certain depth value under the indentation surface. Moreover, the deepness of the plastic deformed region increases with the value of the friction coefficient (Mata and Alcala, 2004; Antunes et al., 2006).
Additional calculations were performed in order to study the friction effect on the values of constraint factor and representative strain of the Vickers indentation of the annealed mild steel studied by Tabor ($E/\sigma_y = 21000/23$; $n = 0.198$). It is assumed that the value of the friction coefficient between well polished metallic surfaces and diamond lies within 0.1 and 0.15 (Tabor, 1951; Giannakopoulos and Larsson, 1997). For the calculations, μ was fixed at 0.15. $H = 1540\,MPa$ and $H = 1651\,MPa$ were found for the frictionless case and for Vickers indentation performed with $\mu = 0.15$, respectively. This results shows that friction increases the hardness by 7%, which is in agreement with the previous results obtained for Vickers indentation (Giannakopoulos et al., 1994; Mata and Alcala, 2004). The procedure given paragraph 2.3 was performed from a trapezoid with vertices defined by the following combinations of elasto-plastic properties: $E = 210000$; $\sigma_y = 160, 250$; $n = 0.15, 0.2$. With the proposed procedure, $\varepsilon_{RP} = 0.0693$ was obtained for Vickers indentation with $\mu = 0.15$. Compared with the frictionless case, for which $\varepsilon_{RP} = 0.0948$ was obtained, it can be concluded than friction has for consequence the decrease in the value of representative strain. This result is in agreement with the decrease in the maximum plastic strain with the increase in the friction coefficient observed by Antunes et al. (Antunes et al., 2006). These values of hardness and representative strain lead to $C_f = 2.755$ and $C_f = 3.135$ for the frictionless case and for Vickers indentation performed with $\mu = 0.15$, respectively. The values of constraint Factor obtained in the case of contact with friction are close to the experimental values obtained by Tabor (1951) and Chaudhri (1998). It is shown that the increase in constraint factor occurs for friction contacts as compared to frictionless ones because of the increase in hardness and decrease in representative strain.

3.2. Martens hardness

3.2.1. Representative strain

It is useful to remind that the representative strain presented in this section is characteristic of the "Martens" hardness in the case of a conical indenter equivalent to the Vickers indenter. This representative strain is thus characteristic of the relationship between the applied load, *F* and the penetration depth, *h*, or relationships between the combinations of these parameters, as for example, the total work-applied load relationship or the total work-penetration depth relationship.
Fig. 10 shows the values of the representative strain calculated from the "Martens" hardness by using Eqs. (7) and (12).

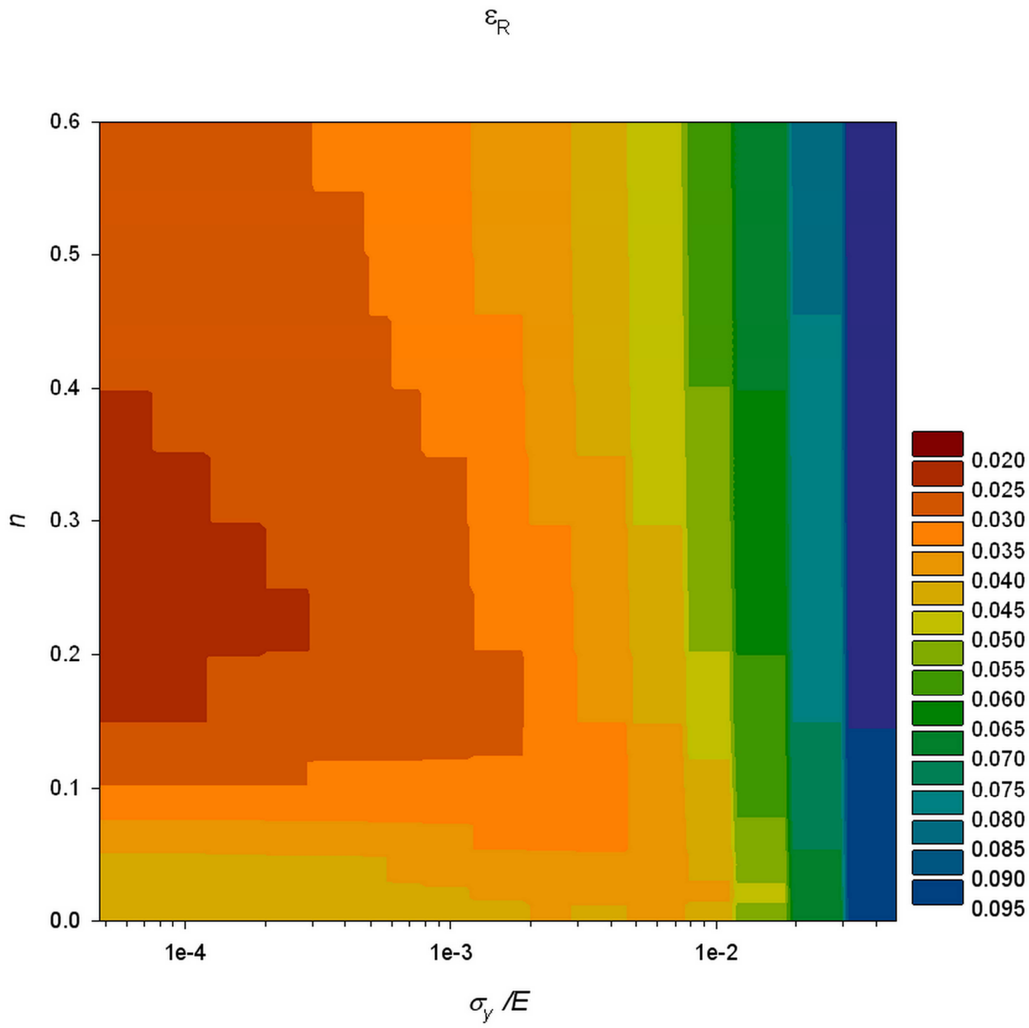


Fig. 10: Representative strain obtained from the Martens Hardness (Totale representative strain)

As for the case of the Mean Pressure, Fig. 10 shows that no universal value for the equivalent strain introduced by a Vickers indentation exists if we consider the “Martens” hardness. For the studied materials, the values of the representative strain obtained from the “Martens” hardness, lie between 0.025 and 0.095 which are very smaller than the values of representative strain obtained from the Mean Pressure which are in the range between 0.09 and 0.26. Chollacop et al. (2003) proposed reverse algorithms using dual sharp indenters to determine stresses for two representative strains and thus stress-strain curves of power law strain hardening materials. The proposed methodology is based on the measurement of a *F–h* curve (or Martens hardness). From this methodology, representative strains in the range between 1.7% and 8.2% were identified for conical indenters of semi-apex angles ranging between 50° and 80°. The comparison between the values of representative strain given in Figs. 8 and 10 shows that a similar procedure based on the measurement of the *F-a* relationship (or mean pressure) should lead to a better prediction of stress-strain curves for large values of plastic deformation.
In order comparing our results to the representative strains proposed by Dao et al. (2001) and Cao and Huber (2006), the “characteristic” strain, ε_{R0}, corresponding to the nonlinear part of the total effective strain accumulated beyond the yield strain was calculated from the total

strain ε_R. Fig. 11 shows the values of ε_{R0} obtained from our procedure for the various studied materials.

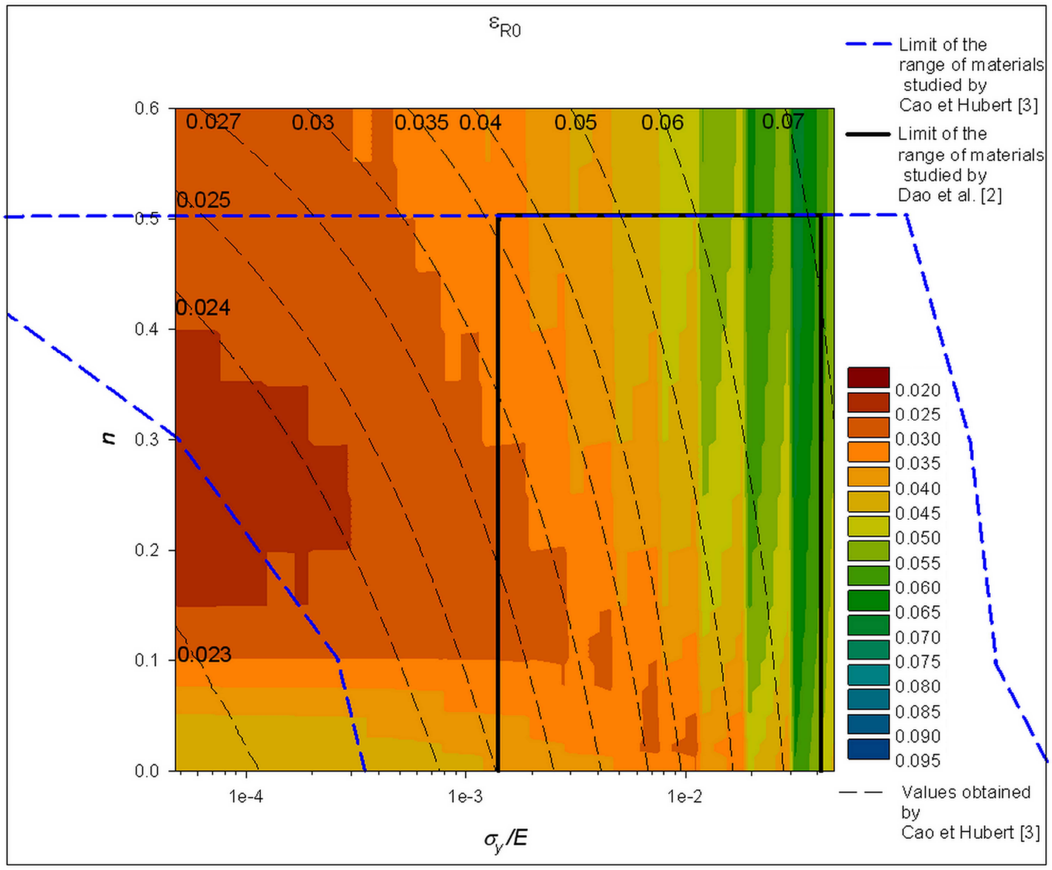


Fig. 11: Representative strain obtained from the Martens Hardness (Representative strain corresponding to the nonlinear part of the total effective strain accumulated beyond the yield strain).

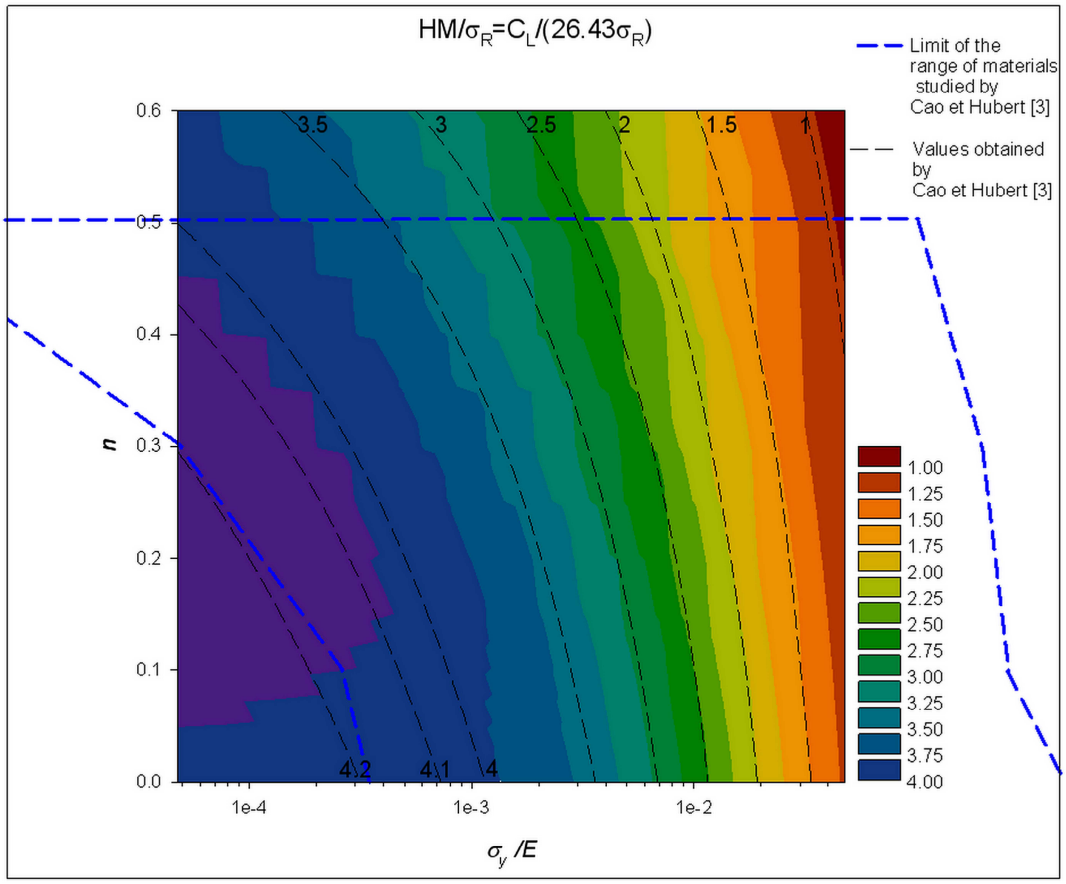
Using dimensional analysis, a representative strain ε_{R0} = 0.033 was identified by Dao et al. (2001). This value lies within the range of representative strains determined from our procedure for the materials studied by Dao et al (2001). The representative strain defined by Dao et *al.* becomes very different from our results for materials of large σ_y / E ratio, for which ε_{R0} is about equal to 0.065. Our results are in accordance with those obtained by Ogasawara et al. (2005) and Cao and Huber (2006). Indeed, in their paper, Ogasawara et al. (2005) argued that the constant representative strain defined by Dao et al. (2001) only works well for a limited range of materials.

In the work of Cao and Huber (2006), it is also found that the representative strain defined by Dao and al. (2001) cannot lead to a one-to-one relationship between the representative stress, indentation loading curvature, and reduced Young's modulus. In order to obtain a one-to-one relationship with good level of accuracy between σ_R/C_L and C_L/E^* for the studied materials, Cao and Huber (2006) proposed to define the representative strain as a function of C/E^* instead of a constant (Eq. (17) in Ref. (Cao and Huber, 2006)). Fig. 11 shows that the values of the representative strain obtained by Eq. (17) in Ref. (Cao and Huber, 2006) are very close to those obtained from our procedure.

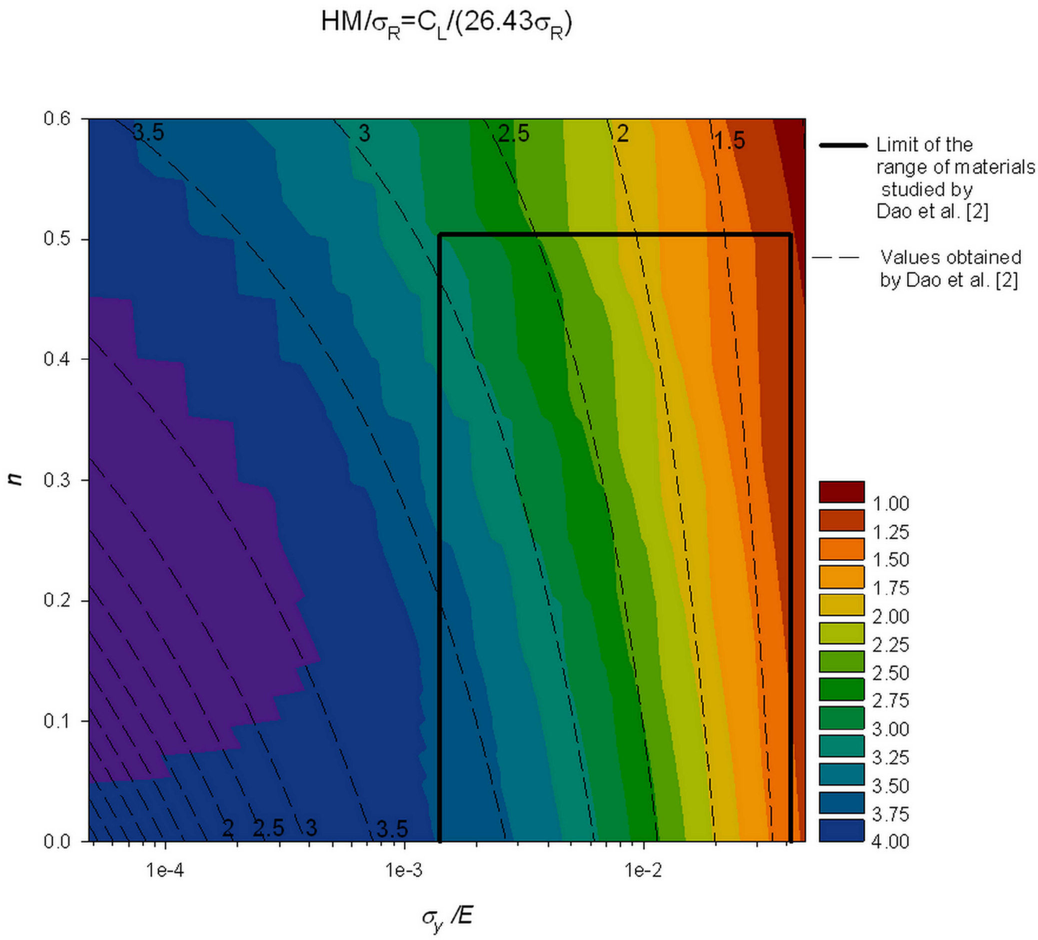
3.2.2. Constraint Factor

The values of the HM/σ_R ratio obtained from Eqs. (2) and (3) and the values of representative strain shown Fig. 10 are given in Fig. 12. The meaning of the HM/σ_R ratio is similar to that of the constraint factor calculated from the mean pressure (H/σ_R).

As for the case of the constraint factor, Fig. 12 shows that the HM/σ_R ratio varies significantly depending on the values of n and σ_y/E. Fig. 12.a shows that the values of the HM/σ_R ratio obtained from Eqs. (11) and (17) in the work of Cao and Huber (2006) are in fairly good agreement with those obtained from our procedure. Despite the difference between the representative deformation identified by Dao et al. and those determined in the presented work, the HM/σ_R ratios proposed by Dao et al. (2001) are also close to our results (Fig. 12.b).



(a)



(b)

Fig. 12: Values of the HM/σ_R ratio obtained from Eqs. (2) and (3) and the values of representative strain. (a): comparison with the values of Dao et al. (2001); (b): comparison with the values of Cao and Huber (2006).

3.3 Discussion about the definition of the representative strain

As mentioned in introduction, the representative strain, ε_R, is a key input parameter for any methodology that attempts to extract plastic properties from Martens hardness or mean pressure. Concerning the methodologies based on the Martens hardness, Dao et al. (2001) have used a dimensional analysis to estimate ε_R, for Vickers indentation as 3.3%. Chollacoop et al. (2003) extended this analysis for conical indenters of different semi-apex angles, θ, and show that ε_R linearly decreases with increasing θ. The experiments of Chollacop and Ramamurty (2005) show that the developed algorithms to assess properties of materials based on the use of these representative strains predict the stress-strain curve with good accuracy. The accuracy of the ε_R values estimated by Dao et al. (2001) and Chollacoop et al. (2003) on one side and by Atkins and Tabor (1965) on the other was compared by Chollacop and Ramamurty (2005). From the experimental values of mean pressure and using the constraint factor, C_F, and ε_R values given by Atkins and Tabor (1965) for different conical indenters, they show that the estimated flow stresses are generally significantly larger than the corresponding flow stresses obtained from uniaxial tensile tests. An examination of Fig. 10 in Atkins and Tabor (1965) also shows that the estimated flow stresses is always larger than the corresponding flow stresses obtained from uniaxial tensile tests. For Chollacop and Ramamurty (2005), this result demonstrates that the ε_R values estimated by Chollacoop et al. (2003) from Martens hardness are relatively more accurate than those proposed by Atkins and Tabor (1965). As it was mentioned by Chollacop and Ramamurty (2005), the choice of the representative strain is not the only reason of the overestimation of the flow stress. They indicate that this overestimation is probably due to the implicit assumption by Atkins and Tabor (1965) that the constraint factor C_F for a given cone angle is independent of prior plastic strain. Indeed, Fig. 9 shows that C_F varies depending on the hardening exponent and the yield stress of the indented material. For the materials studied by Atkins and Tabor (1965), Fig. 9 shows that the constraint factor is in the range between 2.8 and 2.9. These values obtained under frictionless conditions are higher that the C_F values obtained by Atkins and Tabor (1965) from experiments, i.e. $2.54 < C_F < 2.7$. From the results shown by Chollacop and Ramamurty (2005) and Atkins and Tabor (see Fig. 10 in Atkins and Tabor (1965)), we can conclude that the overestimation of the flow stress found by Atkins and Tabor (1965) is mainly due to the underestimation of the constraint factor rather than the accuracy of the representative strain.

Several studies were conducted to find a physical meaning of the representative strain and connect this representative strain with the quantity of plastic strain induced by conical indenters (Branch et al., 2010; Prasad et al.,2011). Prasad et al. (2011) show that the ε_R values given by Chollacoop et al. (2003) and Bucaille et al. (2003) obtained using dimensional analysis are in excellent agreement with those computed as the volume-average strain within the elastic–plastic boundary. Branch et al. (2010) shows that a method based on this average plastic strain accurately predicts the increase in indentation hardness within the plastic zones of both Vickers and Rockwell C indents for both linear and power law strain hardening materials. Some aspects about the connection between the representative strain and the plastic volume-average strain are to be discussed.

In the study of Branch et al. (2010), the prediction of the micro-vickers hardness profile is based on the assumption that the constraint factor is constant independently the degree of the work hardening of the material. Figure 9 shows that this assumption is not true. It can be also noticed that Branch et al. (2010) found that the representative strain depends on the material, i.e. ε_R of 0.052 and 0.035 for a power law strain hardening material (P675 SS) and a linear law strain hardening material (303 SS), respectively. In contradiction with this result, they consider that the representative strain does not depend on the degree of work-hardening of each material. The work hardening of a material which follows a power law model leads however to a “new” material which can be sometimes modelled better as a linear hardening material.

On the other hand, only plastic strains above 0.002 are included in the determination of ε_R in the analysis of Branch et al. (2010) and Prasad et al. (2011). Branch et al. (2010) justify this value because it is consistent with the common definition of the 0.002 offset yield strength. However, FEM results show that a little difference in the value of the offset yield strength has as a consequence a strong difference in the plastic zone size and thus a strong difference in the value of the volume-average plastic strain (Fig. 13). In consequence, the determination of the values of representative strain is subjective if it is obtained from the values of volume-average strain calculated within the plastic zone of the indentation. The representative strain defined as the volume-averaged plastic strain within the plastic zone of the indentation was validated by Branch et al. (2010) from the measurement of the increase in indentation hardness within the plastic zones obtained with both Vickers and Rockwell C. However, because of the high values of plastic strain at locations nearest to the surface, a noticeable variation in the value of the representative strain has only for consequence a small variation in the hardness profile. Therefore, the good agreement between the predicted micro-indentation hardness values and the experimentally measured hardness values cannot validate the representative plastic strain proposed by Branch et al (2010). Lastly, Prasad et al. (2011) show that, contrary to the ε_R values given by Chollacoop et al. (2003), the experimental values of Atkins and Tabor (1965) are considerably higher than those computed as the volume-average strain within the elastic–plastic boundary. From this result, they concluded that the universal definition of representative strain given by Atkins and Tabor (1965) is not valid for conical indentation. In our opinion, the fact that the volume-average strain is closer to the representative strain given by Chollacoop et al. (2003) than to the representative strain given by Atkins and Tabor (1965) does not mean that the representative strain obtained from *F-h* curves (or Martens hardness) is valid and that the representative strain based on the mean pressure is not valid.

As mentioned by Dao et al (2001), the value of the representative strain depends on the choice of functional parameters that were used to describe the indentation process (the Martens hardness or the mean pressure). We demonstrate in this paper that the representative strain also depends on the mechanical properties of the indented material. Further investigations must be performed to better understand the connection between representative strain and physical quantities.

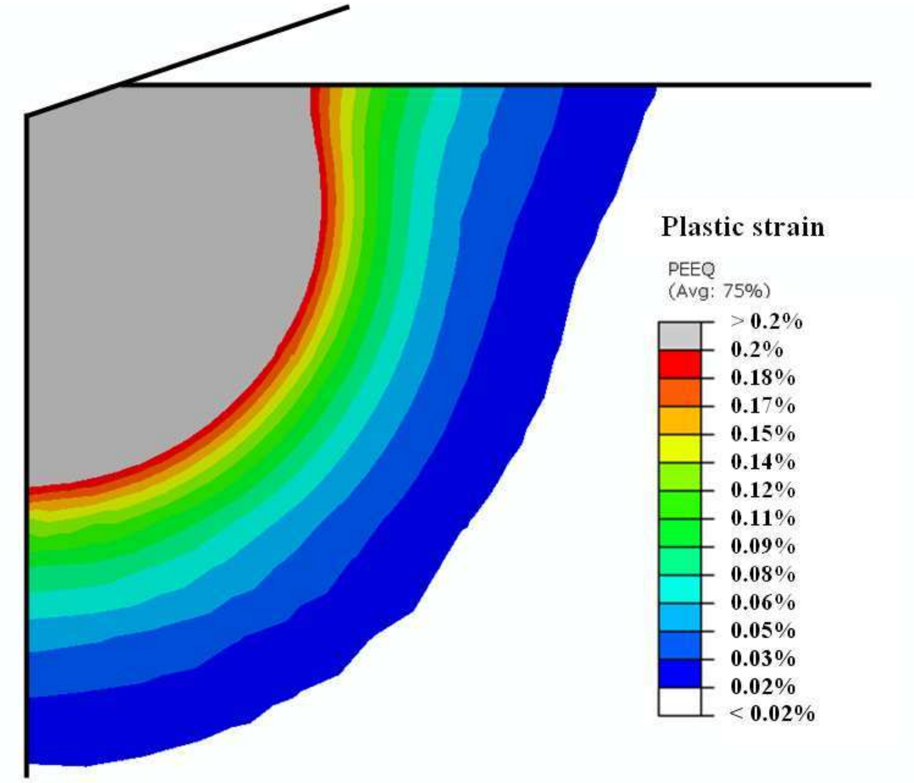


Fig 13: Plastic strain distribution induced by conical indenter equivalent to the Vickers indenter obtained for a material with E=120000 MPa, σ_Y= 3 MPa and n=0.466 (material similar to that studied by Prasad et al. (2011)).

Conclusion

In the present work, an investigation on the definition of the representative strain in conical indentation was carried out. Following a procedure based on finite element simulations of indentation of elasto-plastic materials, news values of representative strain were determined.

Contrary to all previous studies, the proposed procedure is based on no assumption. This procedure leads to the determination of values of representative strains valid for power law strain hardening materials. Two representative strains were defined: the representative strain characteristic of the mean pressure and the representative strain characteristic of the Martens hardness or the indentation loading curvature. The results obtained following our procedure clearly demonstrates that the representative strain, ε_R, is not the same if we consider the Martens hardness, *HM*, or Mean pressure, *H*. For the studied materials which include engineering materials, the values of the representative strain obtained from the "Martens" hardness, lie between 0.025 and 0.095 depending on the material. If the Mean pressure is considered, the values of the representative strain lie between 0.08 and 0.25, depending on the material. These results show that the mean pressure-based representative strain is higher than the Martens hardness-based representative strain. This result means that reverse algorithms using dual sharp indenters based on the measurement of the *F-a* relationship (or mean pressure) should lead to a better prediction of stress-strain curves for large values of plastic deformation. As for the case of representative strain, no constant values of constraint factor H/σ_R and of HM/σ_R ratio were found for the studied materials.

The concept of representative strain and constraint factor, C_F, or Π dimensionless functions was introduced with the objective converting the hardness-strain to the stress-strain curve of the studied material. The ideal situation prevails if these parameters are independent of material being indented. The presented results show that the concept of universal value for the representative strain or the constraint factor introduced by a Vickers indentation doesn't exist.

The representative strain induced by a conical indenter equivalent to the Vickers indenter depends on both studied material and measured quantities (Martens hardness or mean pressure).

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