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GPU Ray-Traced Collision Detection: Fine Pipeline Reorganization

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Abstract: Ray-tracing algorithms can be used to render a virtual scene and to detect collisions between objects. Numerous ray-tracing algorithms have been proposed which use data structures optimized for specific cases (rigid objects, deformable objects, etc.). Some solutions try to optimize performance by combining several algorithms to use the most efficient algorithm for each ray. This paper presents a ray-traced collision detection pipeline that improves the performance on a graphics processing unit (GPU) when several ray-tracing algorithms are used. When combining several ray-tracing algorithms on a GPU, a well-known drawback is thread divergence among work-groups that can cause loss of performance by causing idle threads. In this paper we avoid branch divergence by dividing the ray tracing into three steps with append buffers in between. We also show that prediction can be used to avoid unnecessary synchronizations between the CPU and GPU. Applied to a narrow-phase collision detection algorithm, results show an improvement of performance up to 2.7 times.

1 INTRODUCTION

Collision detection (CD) is an important task in virtual reality (VR). From a set of objects, we need to know those which collide. Nowadays collision is one of the main bottlenecks in VR applications due to the complexity of environments we want to simulate and the real-time constraint required by VR. We need to simulate large environments that can count millions of objects. And we want to include complex objects that can be deformable, undergo topology changes or be fluids.

To handle such a work load, collision is generally divided into two phases: broad-phase and narrow-phase. The broad-phase takes the whole set of objects from the environment and provides a set of pairs of objects that might collide. The narrow-phase takes these pairs and outputs the ones that actually collide. This paper focuses on the narrow-phase which is nowadays the most time-consuming phase.

Recent approaches use GPGPU (general-purpose computing on graphics processing units) to take advantage of the high computational power of recent GPUs. Parallel implementation of the broad-phase sweep and prune algorithm on GPUs improve performance by orders of magnitude (Liu et al., 2010). The narrow-phase can also be implemented on a GPU (Pabst et al., 2010).

In the literature four categories of narrow-phase algorithms are studied, including image-based algorithms that uses rendering techniques to detect collisions. As GPUs are designed especially for rendering, image-based methods are well suited for GPGPU implementations. In particular ray-tracing algorithms can be used and several ones can be combined to improve performance (Lehericey et al., 2013a). When implementing several ray-tracing algorithms on GPUs, high branch divergence in the execution can lead to underuse of the GPU and reduced performance.

Main contributions: we present a new pipeline organization of the narrow-phase for ray-traced collision detection. The goal of our pipeline is to minimize thread divergence on GPUs to improve performance. Our pipeline is designed to be able to integrate several ray-tracing algorithms without causing overhead. We then present a prediction method to avoid CPU-GPU synchronization between steps of the pipeline to further improve performance.

This paper is arranged as follows. The following section provides a review of the relevant literature. Section 3 presents our ray-traced collision pipeline. Section 4 shows how prediction can be used to avoid unnecessary synchronization between the CPU and GPU. Implementation and results are discussed in Section 5. Section 6 concludes this paper.
2 RELATED WORK

This section focuses on the literature on CD which is most pertinent to our work, in particular image-based algorithms for the narrow-phase. For an exhaustive overview on CD, readers should refer to surveys such as [Teschner et al., 2005; Kockara et al., 2007].

2.1 Image-based Collision Detection

Image-based algorithms use rendering techniques to detect collisions. These methods are often adapted for GPUs, which are processors designed for rendering. Image-based methods can be used to perform collision detection on rigid and deformable bodies. These techniques usually do not require any pre-processing steps which makes them suitable for dynamically deforming objects. Image-based algorithms can either use rasterization or ray-tracing techniques.

Rasterization can be used to compute a layered depth image (LDI) which can be post-processed to detect collisions [Allard et al., 2010]. The CInDeR algorithm [Knott, 2003] uses rasterization to implicitly cast rays from the edges of an object and counts how many object faces the ray passes through. If the result is odd then there is collision; otherwise there is no collision. [Heidelberger et al., 2003] take three-dimensional closed objects and output the intersection volume without the needs of any data structures making it suitable for deformable objects. The main drawback of these techniques is the discretization of the space through rendering that induces approximations and may lead to the omission of small objects.

Ray-tracing algorithms can be used to overcome these approximations. [Wang et al., 2012] show that ray tracing can be used to compute a LDI with a higher resolution around small objects by increasing the density of rays locally to increase the precision. [Hermann et al., 2008] detect collisions by casting rays from each vertex of the objects. Rays are cast in the inward direction; the ones that hit another object before leaving the source object detect a collision. Before casting a ray, the ones that are outside the intersection of the bonding volumes are dropped since they cannot collide.

Based on Hermann et al.’s algorithm, [Lehericey et al., 2013a; Lehericey et al., 2013b] introduced an iterative ray-traced collision detection algorithm (IRTCD). This method combines two different ray-tracing algorithms; a full and an iterative algorithm that are respectively used for high and small displacements between objects. The full algorithm can be any existing ray-tracing algorithm. The iterative algorithm is a fast ray-tracing algorithm that exploits spatial and temporal coherency by updating the previous time-step. These algorithms can be used on GPUs for improved performance.

2.2 GPU Computing

Several methods consider the GPU as a streaming processor. As GPUs are massively parallel processors, computations should be arranged to be made parallel [Nickolls and Dally, 2010].

When code runs on a GPU, the same function (called kernel) is called by multiple threads (called work-item) on different data. Work-items do not run independently but in a group (called work-group); in a work-group, work-items run in a single instruction, multiple data way. On a conditional branch, if the work-items diverge among a work-group, each branch runs serially. This phenomena is called thread divergence and reduces performance [Zhang et al., 2010].

One case of thread divergence is having a sparse input, in which case some work-items have nothing to do and are idle in a work-group while the other work-items finish. Collision-streams [Tang et al., 2011] use stream compaction to avoid kernel execution on sparse inputs. Collision detection is divided into several steps and streaming compaction is used between steps to ensure dense input for each kernel.

2.3 Synthesis

Ray-traced collision detection can be used on GPUs to achieve high performance on complex scenes. In a simple scenario only one ray-tracing algorithm can be used to resolve collisions. But in complex scenes with objects of different nature we can employ several ray-tracing algorithms to optimize each nature of object. With rigid objects we can use algorithms with complex data structures. With deformable objects we need data structures that can be updated at each time-step. In the case of topology changes we need to reconstruct the ray-tracing data structures occasionally. Iterative ray-tracing can be used to accelerate all of these algorithms.

In complex scenes where all these properties are represented, combining several ray-tracing algorithms improves performance. In such situations, the need of a narrow-phase that can easily manage diverse ray-tracing algorithms without losing performance emerges.
3 RAY-TRACED CD PIPELINE

To perform our ray-traced CD method there are several ray-tracing algorithms available in the literature. We selected three algorithms. We qualify the two first ones as ‘full’ ray-tracing algorithms; these algorithms can be used at any time. The third algorithm, called iterative, requires specific conditions.

The first full algorithm is a stackless bounding volume hierarchy (BVH) traversal. This algorithm uses an accelerative structure and can be used for rigid objects. The second full algorithm is basic ray-tracing. This algorithm does not use any additional accelerative structures and can be used for deformable objects. If we used the stack-less BVH traversal algorithm or deformable objects, we would need to update the accelerative structure at each time-step.

The third ray-tracing algorithm which we do not qualify as ‘full’ is the iterative ray-tracing algorithm. This algorithm is incremental and updates the previous time-step. This algorithm cannot be used at any time because it needs an extra input to work; this extra input is the reference of the last hit triangle in the previous time-step (called temporal data in the remainder of this document). The iterative algorithm needs a full ray-tracing algorithm to first create temporal data and then it can be used to replace the full algorithm in the following steps. We can use the iterative algorithm as long as relative displacement between the ray and target of the ray remain small. When temporal data is available, the iterative algorithm is faster than the two previous ones and should be preferred.

Before executing the ray-tracing we can perform simple culling tests to reduce the total number of rays to cast. If the origin of the ray is outside the bounding volume of the target, we can remove that ray because it cannot be inside the target. For rays using the iterative algorithm, we test if temporal data is available from the last step. If no temporal data is available we can drop the vertex because the iterative algorithm requires temporal data. The absence of temporal data indicates that at the previous time-step no collision was detected for this vertex and no prediction indicated a possible future collision.

When combining several ray-tracing algorithms on GPUs, the naive implementation consists of executing one large kernel that implements all the ray-tracing algorithms with a switch to select which algorithm to use for each vertex. This naive implementation will cause thread divergence for two reasons. First, all the ray-tracing algorithms are executed by the same kernel; this adds a branch divergence on the execution of each ray trace. Each work-group that executes more than one algorithm will execute them sequentially. Second, in a context where we can safely cull some vertices to reduce the amount of computation, this naive implementation will initiate work-items that finish immediately. These work-items will then be idle while the other work-items of the work-group finish.

This section presents our pipeline distribution of tasks to improve the performance of ray-traced collision detection on GPUs. Subsection 3.1 gives an overview of the pipeline and the three steps. Subsection 3.2 and 3.3 detail the three steps: per-pair, per-vertex and per-ray steps.

3.1 Pipeline Organization

To achieve collision detection our narrow-phase pipeline must perform several tasks: **Apply a measurement of displacement** on the pair of objects to decide if we use a full or the iterative ray-tracing algorithm. **Cull vertices** with simple criteria to avoid unnecessary ray-tracing. **Execute the ray-tracing** from the remaining vertices; the algorithm depends on the nature of the object or if we decided to use the iterative algorithm.

The displacement measurement can be applied globally on a pair of rigid objects and needs to be applied per vertex when a deformable object is present to take into account internal deformations. Pairs (or vertices) classified with small displacement will use the iterative ray-tracing algorithm.

Vertex culling uses a drop criterion to remove vertices that do not need to be tested with ray-tracing because a simple test can discard collision. For the vertices classified as evidencing displacement, we check if the vertex is inside the intersection of the bounding volumes of the two objects of the pair. For the vertices classified as showing small displacement we test the presence of temporal data.

Ray-tracing is executed for each of the remaining vertices. We use the iterative algorithm when selected; otherwise, we use an algorithm that depends on the nature of the target of the ray-tracing.

These tasks show that the narrow-phase must process data at three different levels of granularity: per-pair, per-vertex and per-ray granularity. The goal of this decomposition is to avoid branch divergence and ensures a dense input at each step. Figure 1 gives an overview of the pipeline organization.

Append buffers are used between each step to store data. The kernel preceding the append buffer fills it. Each entry of the buffer generates threads in the following kernel. The main purpose is to guarantee a dense input to the following kernel and avoid idle threads.
Figure 1: Pipeline organization of the narrow-phase. The narrow-phase starts from the list of pairs of objects detected by the broad-phase. The first step is applied on the pair and we separate them into three categories: rigid pairs with small displacement, rigid pairs with high displacement and deformable pairs. The second step is applied on the vertices of the objects of the pairs; we select which ray-tracing algorithm to use depending on the nature of the objects and the result of the displacement measurement. The last step is applied on the rays and is the execution of the ray-tracing algorithms.

In the following subsections 3.2, 3.3 and 3.4 we detail the content of the three steps.

### 3.2 Per-Pair Step

The per-pair step starts from the list of pairs of objects provided by the broad phase. In this step one thread corresponds to one pair.

First we split the pairs into two categories: those which contain only rigid objects and those which contain deformable objects. This separation is applied because the criterion to decide if we use a full or the iterative algorithm is different.

For the pairs with only rigid objects we apply at this stage a displacement criterion to separate those with high displacement that will use a full ray-tracing algorithm and the ones with small displacement that will use the iterative algorithm. We cannot apply a displacement criterion on the pairs that contain at least one deformable object in this stage. In such cases displacement needs to be measured locally to take into account internal deformations.

### 3.3 Per-Vertex Step

The per-vertex step starts from each list of pairs constructed in the per-pair step. In this step one thread is executed for each vertex of each object of each pair.

For the pair containing deformable objects, we can now apply a displacement measurement on the vertices to separate the ones with small displacement that will use the iterative algorithm and the others that will use a full algorithm.

Then we apply a drop criterion on the vertices to cull the ones that can be safely discarded. The criterion is different for vertices using a full algorithm and vertices using the iterative algorithm. For the vertices that use a full algorithm, we check if the vertex is inside the intersection of the bounding volumes of the two object of the pair. If not, we can drop this vertex as it cannot collide. For the vertices that use the iterative algorithm, we test if temporal data is available from the preceding step. If no temporal data is available we can drop the vertex.

After applying the drop criterion, each remaining vertex generates a ray that will be cast on the other object of the pair; the parameters needed to cast these rays are stored in three buffers. Vertices using the iterative algorithm fill the list of rays for the iterative ray-tracing algorithm. Rays generated from vertices using a full algorithm need to be separated into two categories: rays cast to a rigid object use a stackless BVH ray-tracing algorithm; rays cast to a deformable object use the basic ray-tracing algorithm.

### 3.4 Per-Ray Step

The per-ray step starts from each list of rays and executes the ray-tracing. Each thread casts one ray.

The BVH ray list uses the stackless BVH traversal algorithm. This algorithm uses a BVH as an accelerating ray-tracing structure (Wald et al., 2007) with a stackless traversal adapted for GPUs (Popov et al., 2007). The basic ray list uses the basic ray-tracing algorithm. This algorithm does not use any accelerating structure; for each ray we iterate through each triangle. This method has high complexity but does not necessitate any accelerating structures that would need to be updated after each deformation. The iterative ray list uses the iterative ray-tracing algorithm (Lehericey et al., 2013a). This algorithm can be used on both rigid and deformable objects.

Each ray-tracing algorithm is implemented and executed in a separate kernel to avoid branch divergence in the GPU. The ray-tracing algorithms output contact points for the physics response.
4 BUFFER COMPLETION PREDICTION

Along the narrow-phase we need to know the number of elements contained in each append buffer (which we will refer to as completion in the rest of this section). For each append buffer we need to know a priori the completion to allocate sufficient space and a posteriori to know how many threads must be launched in the following kernels. But these completion values are unavailable a priori and stored in the GPU memory a posteriori. To avoid extra computations or synchronizations that would lower the performance, we can rely on predictions.

In this section we will first explain, in subsection 4.1, our general prediction scheme and then we will apply this prediction scheme to the append buffer sizes and on the workload in subsection 4.2 and 4.3.

4.1 Generic Prediction

Let $N_t$ be the number of elements contained in an append buffer at the current time-step, and $N_{t-1}$ at the previous time-step. $p(N_t)$ is the predicted value at the time-step $t$.

We have access to $N_{t-1}$, i.e. the exact count that should have been used in the previous time-step. This value can be retrieved asynchronously with no extra cost. To predict the evolution of the value, we rely on the evolution of the number of pairs issued by the broad-phase which gives an indication on the evolution of the global workload. Let $nbPairs_t$ (and $nbPairs_{t-1}$) be the number of pairs detected by the broad-phase at the time-step $t$ (and $t-1$).

Equation 1 gives a prediction value $p(N_t)$. We take as input the real value at the previous time-step, $N_{t-1}$. We weight the value with the evolution of the number of pairs that give us an indication of the evolution of the workload. And we multiply the result with a confidence level of at least 1 to take into account variability.

\[ p(N_t) = N_{t-1} \times \frac{nbPairs_t}{nbPairs_{t-1}} \times confidence \] (1)

At the end of the time-step we can asynchronously retrieve the real value $N_t$ and compare it with the prediction $p(N_t)$ to check if the prediction was high enough. If the prediction was too low we have two strategies. The simplest one is to ignore the error and continue the simulation. This solution will cause errors in the simulation but will be suitable for real-time applications. The second solution is to backtrack the simulation and use the real values. This will achieve a correct simulation and will be preferred for offline simulations.

4.2 Buffer Size Prediction

Before we launch a kernel that will fill an append buffer we need to know a priori the number of elements that will be inserted to allocate sufficient space. To get this number we could execute the kernel twice: once to get the number of outputs and once to fill the append buffer.

We can use prediction to avoid the first execution. We apply the prediction equation to evaluate the size
of the append buffer. We can use a large value of confidence in these predictions as it does not affect computation size but does affect buffer size.

4.3 Workload Prediction

To be able to schedule a kernel that takes as input an append buffer we need to know the completion of the buffer to know how many threads to launch. This completion value is in the GPU memory and the kernel execution is scheduled by the CPU. We have two solutions to manage this data dependency:

The first solution, called exact workload, consists of reading back the completion from the GPU to the CPU. Then we can schedule the next kernels with the exact number of threads. The upper part of Figure 2 shows the timeline of the events on the CPU and GPU. The major drawback of this method is the synchronization between the CPU and GPU; the GPU is idle while waiting for the next kernel’s execution.

The second solution, called workload prediction, does not rely on the exact completion, but predicts it. To predict the completion we can apply the prediction equation with a large value of confidence to minimize the risk of missing some rays. If we run too many threads, they will immediately exit after checking if their index exceed the completion value stored in the GPU. The drawback of this method is that large prediction will lead to empty threads in the GPU, but these empty work-items will be all grouped in the last work-groups and thus will not provoke branch divergence. The advantage of this solution is that it does not trigger any synchronization between the CPU and GPU as shown in the bottom part of Figure 2.

5 PERFORMANCE EVALUATION

This section presents our experimental environment and our performance results. First, in 5.1 we present our experimental setup. Then in subsection 5.2 we show the performance improvement when using our ray-traced pipeline method. In subsection 5.3 we show the performance gain obtained by using prediction. Finally in 5.4 we measure the quantity of errors introduced by the predictions.

5.1 Experimental Setup

To measure the performance improvements of our ray-traced narrow-phase we measured the GPU computation time of 10 second simulations executed at 60 Hz on two different experimental scenes: ‘rigid objects’ and ‘rigid and deformable objects’.

Figure 3: The four rigid objects used in the simulation respectively composed of 902, 2,130, 3,264 and 3,458 triangles.

In the scene ‘rigid objects’ (see Figure 4) 1,000 rigid non-convex objects fall on an irregular ground. Four different objects presented in Figure 3 are used in equal quantities. Up to 10,000 pairs of objects are tested in the narrow-phase.

Figure 4: ‘Rigid object’ scene: 1,000 concave objects fall on an irregular ground.

In the scene ‘rigid and deformable objects’ (see Figure 5) 216 rigid non-convex objects fall on an irregular ground and 36 deformable sheets fall on the top of them. The rigid objects are the same as in the first scene. The deformable sheets are square shaped and composed of 8,192 triangles. Up to 860 pairs of rigid objects and 600 pairs of rigid/deformable objects are tested in the narrow-phase.

Figure 5: ‘Rigid and deformable object’ scene: 216 concave objects fall on an irregular ground and 36 deformable sheets fall over them.

Our experimental scenes were developed with bullet physics 3.x using a GPU. Our narrow phase were implemented for GPU with OpenCL. All scenes were executed using an AMD HD 7990.

5.2 Pipeline Performance

To measure the speedup obtained with our narrow-phase pipeline, we compared the GPU computation
time of the narrow-phase while using our pipeline reorganization and without it.

Figure 6: Performance comparison of GPU collision detection with and without our pipeline reorganization in the scene ‘rigid objects’. With only a full ray-tracing algorithm our method gives an average speedup of 1.31 times and with the combination of a full and iterative ray-tracing algorithms our methods gives an average speedup of 2.73 times.

In the scene ‘rigid objects’ we measured the performance of our pipeline reorganization when using only the full ray-tracing algorithm (BVH ray traversal) and when using both full and iterative ray-tracing algorithms. Result are shown in Figure 6.

The first result is the speedup obtained when using our pipeline method with only the full ray-tracing algorithm. We get an average speedup of 1.31 times when using our pipeline ray-tracing method. This speedup can be explained by the per-vertex step that guarantees that the ray-tracing step is executed on dense data even if the input of the per-vertex step is sparse.

The second result is the speedup when using our pipeline method with the combination of full and iterative ray-tracing algorithms. The average speedup when using our pipeline ray-tracing method is 2.73 times. The extra speedup compared to the previous case come from the execution of each ray-tracing algorithm in a different kernel in the per-ray step which reduces branch divergence in the GPU.

In the scene ‘rigid and deformable objects’ we measured the performance of our pipeline reorganization when three ray-tracing algorithms are used. Result are shown in Figure 7. In this scene we get an average speedup of 1.84 times when using our pipeline ray-tracing method.

5.3 Workload Prediction

To show the performance improvement when using our workload prediction strategy, we compared the GPU computation time of simulation using the workload prediction and the exact workload strategies.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Average gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Rigid objects’ Only full ray-tracing</td>
<td>2.84 ms</td>
</tr>
<tr>
<td>‘Rigid objects’ Full and iterative ray-tracing</td>
<td>1.97 ms</td>
</tr>
<tr>
<td>‘Rigid and deformable objects’ Full and iterative ray-tracing</td>
<td>2.05 ms</td>
</tr>
</tbody>
</table>

Table 1: Average GPU performance gain between the exact and prediction strategies for the workload. The workload prediction strategy is up to 2.84 ms faster per time-step than the exact workload strategy.

Table 1 gives the average gain when using the workload prediction against the exact workload strategy. We measured the gain on both ‘rigid object’ and ‘rigid and deformable objects’ scenes. For the ‘rigid object’ scene we measured the gain when using only the full ray-tracing algorithm and when using both full and iterative ray-tracing algorithms. The results show that the workload prediction strategy is between 2.0 and 2.8 ms faster per time-step than the exact workload strategy in both cases. This gain comes from elimination of CPU/GPU synchronizations between steps that eliminate the fixed cost (the cost of a CPU/GPU synchronization does not depend on the quantity of work). This reduction is not negligible in a real-time context where we only have a few ms to compute collisions.

5.4 Prediction Error Evaluation

To evaluate the efficiency of our generic prediction equation (Eq. 1), we measured the percentage of missed rays caused by underestimation of the predictions. We compared this percentage of missed rays with a simple prediction equation (Eq. 2) that does not take into account the evolution of the number of pairs detected by the broad phase. In both cases we used a
2\% variability confidence ($\text{confidence} = 1.02$).

\[ \text{buffer}_t = \text{buffer}_{t-1} \times \text{confidence} \]  

(2)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Error reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Rigid objects' Full and iterative ray-tracing</td>
<td>79%</td>
</tr>
<tr>
<td>'Rigid and deformable objects' Full and iterative ray-tracing</td>
<td>28%</td>
</tr>
</tbody>
</table>

Table 2: Measurement of the reduction of the number of missed rays. Taking into account the evolution of the number of pairs detected by the broad-phase reduces the number of missed rays by up to 79\%.

Table 2 shows the reduction of the errors when using our prediction method against the simple one. We did not include the scene ‘rigid objects’ with only the full ray-tracing algorithm because with only one algorithm the percentage of error is too small to be significant (less than 0.1\%). Results show that taking into account the evolution of the number of pairs reduce the percentage of errors by 28 and 79\%.

6 CONCLUSIONS AND FUTURE WORK

We have presented a pipeline reorganization to improve the performance of ray-traced collision detection using a GPU. Our method enables the integration of different ray-tracing algorithms in our narrow-phase to efficiently handle objects of different nature. By dividing the narrow-phase into three steps, we were able to maintain a dense input throughout the whole pipeline to maximize the GPU cores usage. Our implementation shows an average speedup up to 2.73 times.

In future work we want to extend our narrow-phase pipeline to objects of other nature such as topology changes and fluids. We also want to improve performance by generalizing our pipeline to multi-GPU and hybrid CPU/GPU architectures.

REFERENCES


