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- Design of a SIMO architecture (single input multiple output) for transhorizon radio links
- Selection of the most effective receive antennas in a set of NA possible candidates
- Outage capacity reaching a value of 2.23 bps/Hz significantly superior to standard performances

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Antenna selection in a SIMO architecture for HF radio links

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Abstract This work takes place in the global design of a SIMO architecture (single input multiple output) for transhorizon radio links, aiming at a significant increase in the data rate when compared with standard modems based in general on a SISO scheme (single input single output). The project is subject to available space constraints at the receive end, involving mobile stations or onboard implementation. We consider solutions that appear as extensions of the compact and heterogeneous antenna array that we proposed previously: collocated antennas of different types are set up with the same phase center and present diversity in their polarization sensitivities to make array processing effective. Given the number NC of receive channels, we address the problem of selecting the most effective antennas in a set of NA possible candidates including monopoles, dipoles, loop antennas with various geometries, and orientations. The criterion to be maximized is the SIMO outage capacity, a quantity based on the statistical distribution of the SIMO Shannon capacity estimated for a large number of ionospheric channel realizations, each of them being quantified by its channel impulse response including the receive antenna directional responses. Results are presented in the context of a 1×2 SIMO structure: the identification of the two most effective antennas in a set of $NA = 15$ sensors indicates that the optimal structures involve two orthogonal horizontal dipoles or two vertical orthogonal loop antennas. In these conditions, the outage capacity reaches up to 2.23 bps/Hz, a value that significantly exceeds the performances of standard modems.

1. Introduction

Multiple-antenna systems have been studied extensively in the context of transmission in the UHF and SHF bands, but few papers are dedicated to applications in the HF band and ionospheric radio links. Recently, an article by *Jin et al.* [2014] addressed the feasibility of implementing a cooperative SIMO (single input multiple output) system for transhorizon transmission exploiting receivers distributed within a wide area. On the contrary, the project presented in the following is subject to available space constraints at the receive end, involving mobile stations or onboard implementation. The starting point of this study is the compact and heterogeneous antenna array described by *Erhel et al.* [1998]: collocated antennas of different types are set up with the same phase center and present diversity in their polarization sensitivities to make array processing effective.

As a first step, the channel impulse response is computed for numerous configurations of radio links in a deterministic approach. This calculation combines a model of electron density function of altitude, a ray tracing software, and the directional gain of the receive antenna which depends on the angles of arrival and the polarization characteristics of the incident waves. For a given radio link, the channel transfer function, transposition of the impulse response in the frequency domain, acts as the predominant element in the expression of Shannon capacity.

The project aims at identifying the NC most effective antennas in a set of NA possible candidates including monopoles, dipoles, loop antennas with various geometries, and orientations. The criterion to be maximized is the SIMO outage capacity, a quantity based on the statistical distribution of the SIMO Shannon capacity estimated for a large number of ionospheric channel realizations, each of them being quantified by its channel transfer function. In a 1×2 SIMO scheme, the two most effective antennas in a set of 15 sensors turn out to be either a pair of orthogonal horizontal dipoles or a pair of vertical orthogonal loop antennas, both solutions presenting equal performances. In these conditions, the outage capacity reaches up to 2.23 bps/Hz, a value that significantly exceeds the performances of standard modems.

2. Calculation of the Channel Impulse Response

2.1. Propagation Parameters

For a given point to point radio link, a channel model requires the estimation of significant parameters for the different paths: path loss, elevation angle, and group delay. This estimation is provided by VOACAP which is a well-known reliable tool for HF circuit analysis described in *Sweeney et al.* [1993]. Restricted to midlatitudes, the simulations use the method #25. The anisotropy of the ionosphere is not rigorously taken into account; however, in a simplified approach, we consider that the propagation of an X mode is similar to the propagation of an O mode with a frequency shift equal to a half gyrofrequency. Furthermore, Doppler shifts and temporal fading are not considered in this description. For a given receiving site, VOACAP method #25 operates with a set of five input parameters: link range, azimuth of the transmitter, carrier frequency, date, and hour. The transmit antenna is supposed to be isotropic. The outputs are the number NS of identified propagation modes and, for each of them, the attenuation, group delay, and elevation of the incident wave. Given the receive antenna, the computation of the channel impulse response needs, in addition, the calculation of the antenna spatial response.

2.2. Spatial Response of a Receiving Antenna

In this step, the estimation of the polarization characteristics at the ionosphere exit is of major interest. The incident waves are elliptically polarized in general with a description based on two parameters: polarization ratio R , the modulus of which quantifies the respective lengths of the two axes of the ellipse along which the electrical field rotates and inclination α between the ellipse main axis and the local horizontal. At the exit of the ionosphere, the electron density tends to zero as does the collision frequency (collisions between electrons and neutral molecules): these assumptions are called Budden conditions. Denoting B_L and B_T the components of the terrestrial magnetic field at the exit of the ionosphere, respectively, longitudinal and transverse components relatively to the direction of the propagation vector, the polarization ratio of the incident wave is expressed as

$$R_{\pm} = \frac{i}{2Y_L} \left\{ Y_T^2 \pm [Y_T^4 + 4Y_L^2]^{1/2} \right\} \quad (1)$$

where $Y_L = \frac{qB_L}{m\omega}$, $Y_T = \frac{qB_T}{m\omega}$, q and m are the electron charge and mass, and ω is the carrier frequency.

The plus or minus sign depends on the polarization type (ordinary O or extraordinary X) associated with the incoming wave: the convention is plus sign for X modes and minus sign for O modes.

Consequently, the polarization characteristics depend on the receiver location and the direction of arrival (DOA) identified by a couple of angles, the azimuth and elevation angles $\theta = (Az, El)$. Then, the antenna spatial response is computed with the Numerical Electromagnetics Code software (NEC-2-D). Based on the method of moments, it is suitable for structures described as a mesh of wires or surfaces in case of free space propagation over the ground. NEC considers incident waves with right or left circular polarizations. It has been modified to support elliptical polarizations with parameters R_{\pm} and α estimated in the previous stage. Finally, the antenna spatial response is a complex valued gain $F(\theta, P)$ depending on the DOA θ and the polarization type P (O or X). Its complex-valued nature is related to the elliptical structure of the polarization that is described by *Erhel et al.* [2004] as a phasor vector with real and imaginary components.

2.3. Channel Impulse Response

The combination of ray tracing and antenna gain computation gives the expression of a channel impulse response $h_i(t)$ including the receive antenna identified with index i in a set of NA sensors:

$$h_i(t) = \sum_{k=1}^{NS} A_k \delta(t - \tau_{gk}) F_{ik}(\theta_k, P_k) \quad (2)$$

where NS is the number of identified paths or modes, A_k is the amplitude of mode k (depending on the corresponding attenuation), τ_{gk} is the group delay of path k , and $F_{ik}(\theta_k, P_k)$ is the gain of antenna i for path k (with DOA θ_k and polarization type P_k O or X). For further exploitation, the temporal samples of $h_i(t)$ are saved in a vector \underline{h}_i which contains a number NS of nonzero elements. The channel complex gain $\underline{Hc}_i(f)$, function of frequency f and defined as the Fourier transform of $h_i(t)$, is computed through the FFT: $\underline{Hc}_i(f) = \text{FFT}(\underline{h}_i)$.

3. Obtaining a Large Number of Trials

The criterion for antenna selection requires statistics of SIMO ionospheric channels. It is then necessary to prepare a collection (with a large number of trials) of channel impulse responses or channel complex gains for each group of receive antennas (with a fixed location) under test.

To reach this goal, the parameters of the simulations to be adjusted are as follows:

1. the year: chosen in an interval with solar activity indices varying from a low to a high value; consequently, three different years are considered: year 1954 with a very low solar activity, year 1969 assumed to be representative of a "mean" activity, and year 1958 for a high solar activity;
2. the month in the selected year: four months are selected, corresponding to the four seasons;
3. the hour: one simulation is carried out every hour;
4. the range of the radio link, expanding from 300 km to 1500 km with a step of 300 km;
5. the azimuth of the link, varying from 0° to 360° with a step of 15°; and
6. the carrier frequency varying from 3 MHz to 15 MHz with a step of 3 MHz.

The maximum number of trials is then equal to 172,800. However, each configuration does not generate an operating link as it is observed if no ray propagates from the transmitter to the receiver or if rays exist, but with a prohibitive path loss. Taking these constraints into account and for given year, distance, and frequency, the number of validated trials N_{tr} varies from several hundred to some thousands (2000 typically).

4. Definition of the Outage Capacity

An overview of performance improvements resulting from the implementation of antenna arrays in wireless system has been proposed by *Khalighi et al.* [2002]. This paper specifies, in particular, the expression of Shannon capacity for various schemes in diversity such as SIMO, multiple input single output, and multiple input multiple output (MIMO). In this project, the antenna selection is based on the maximization of the outage capacity of SIMO channels that involves the histogram of the Shannon capacity estimated for each of the N_{tr} valid trials as described by *Proakis* [1995]. These notions are specified in the following for different schemes: nondispersive SISO (single input single output) channel, nondispersive SIMO channel, and finally dispersive SIMO channel case that corresponds to transhorizon radio links.

4.1. SISO, Nondispersive Channel

In this first scheme, the channel impulse response for the trial with index nr is reduced to a single nonzero element denoted $h_{ref}(nr)$: the channel has a flat frequency response. The Shannon capacity, (calculated in a bandwidth equal to 1 Hz), is expressed as

$$C_{siso}(nr) = \log_2 \left(1 + \frac{P_e \cdot |h_{ref}(nr)|^2}{N_o} \right) \quad (3)$$

where P_e is the transmitted power in this frequency band and N_o is power spectrum density of the noise.

The outage capacity is defined as the threshold exceeded by the Shannon capacity with a probability $1 - \varepsilon$, ε being a given value ($\varepsilon = 10^{-1}$ generally).

$$C_{outsiso, \varepsilon} = \sup_{C \geq 0} \{ C : p[C_{siso} < C] \leq \varepsilon \} \quad (4)$$

This criterion is pertinent as it involves a large number of trials to estimate the histogram of C_{siso} and as it takes into account a kind of quality of service.

4.2. SIMO, Nondispersive Channel

In this scheme, the NC channel impulse responses are reduced to single nonzero elements; these coefficients are stored in a $NC \times 1$ column vector for each trial with index nr :

$$\underline{h}(nr) = \begin{pmatrix} h_{ref}(nr) \\ h_2(nr) \\ \dots \\ h_{NC}(nr) \end{pmatrix}$$

where $h_i(nr)$, $i = 1, \dots, NC$ is the gain (for the trial with index nr) for the channel linking the transmitter to the receive antenna with index i .

Table 1. Antenna Description

Antenna Index	Antenna Description
#1	Vertical north-south oriented loop antenna (octagonal shaped; typical size of 1 m)
#2	Vertical east-west oriented loop antenna (octagonal shaped; typical size of 1 m)
#3	Horizontal loop antenna (octagonal shaped; typical size of 1 m)
#4	East-west oriented dipole antenna (typical length of 2 m in a vertical plane)
#5	North-south oriented dipole antenna (typical length of 2 m in a vertical plane)
#6	Vertical passive dipole antenna (12 m length)
#7	Vertical passive monopole antenna (12 m length)
#8	Vertical east-west oriented V-shaped passive dipole (2 × 15 m long elements)
#9	Vertical north-south oriented V-shaped passive dipole (2 × 15 m long elements)
#10	Vertical east-west oriented, two oblique elements dipole (2 × 15 m long, inclination of 45°)
#11	Vertical north-south oriented, two oblique elements dipole (2 × 15 m long, inclination of 45°)
#12	Horizontal north-south oriented dipole (2 × 2 m long linear elements)
#13	Horizontal east-west oriented dipole (2 × 2 m long linear elements)
#14	Combination of two vertical loops NS + j^*EW (matched to circular polarization)
#15	Combination of two vertical loops NS - j^*EW (matched to circular polarization)

The corresponding expression of the Shannon capacity is

$$C_{\text{simo}}(nr) = \log_2 \left(1 + \frac{P_e \cdot \| \underline{h}(nr) \|^2}{N_o} \right) \quad (5)$$

and the benefit of the SIMO solution (array gain) appears through an increase in the signal-to-noise ratio (SNR) as $\| \underline{h}(nr) \|^2 > |h_{\text{ref}}(nr)|^2$.

4.3. SIMO, Dispersive Channel

This scheme corresponds to the ionospheric channel with NC impulse responses presenting a delay spread. Each of them is transposed in the frequency domain (channel complex gain) with N_f frequency bins identified with index nf . For the trial with index nr and the frequency bin with index nf , the NC channel gains are stored in a column vector:

$$\underline{Hc}(nf, nr) = \begin{pmatrix} Hc_{\text{ref}}(nf, nr) \\ Hc_2(nf, nr) \\ \dots \\ Hc_{\text{NC}}(nf, nr) \end{pmatrix}$$

The corresponding expression of the Shannon capacity is, for one frequency bin nf ,

$$C_{\text{simo}}(nf, nr) = \log_2 \left(1 + \frac{P_e \cdot \| \underline{Hc}(nf, nr) \|^2}{N_o} \right) \quad (6)$$

The benefit of the SIMO solution is expressed in terms of array gain (as previously) and diversity gain in addition with values of $|\underline{Hc}_i(nf, nr)|$ that may be superior to $|Hc_{\text{ref}}(nf, nr)|$.

Keeping in mind that the definition of capacity refers to a 1 Hz wide band, the global Shannon capacity in the case of dispersive SIMO channels is expressed as a value averaged on N_f bins:

$$C_{\text{simo LB}}(nr) = \frac{1}{N_f} \sum_{nf=1}^{N_f} C_{\text{simo}}(nf, nr) \quad (7)$$

In this scheme, the outage capacity is defined as

$$C_{\text{outsimo } \epsilon} = \sup_{C \geq 0} \{ C : p[C_{\text{simo LB}} < C] \leq \epsilon \} \quad (8)$$

In the project, combinations of NC antennas among a set of NA possible sensors are then considered and, for each of them, the outage capacity is calculated. The final selection is based on the SIMO/SISO gain

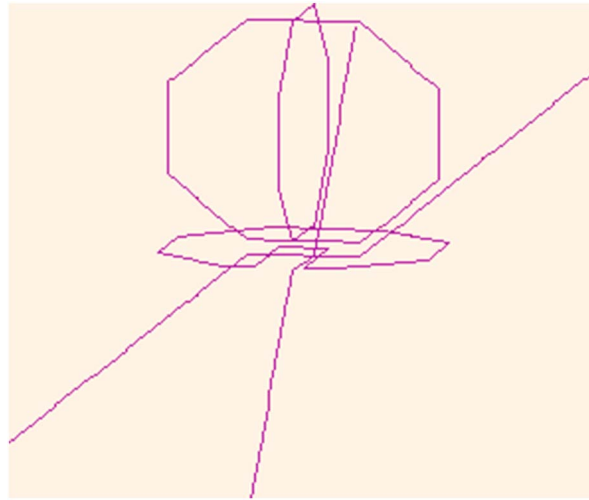


Figure 1. Array of five collocated antennas; the three loop antennas have a diameter equal to 1 m; the two dipole antennas have a length of 2 m.

$G_{\text{out}} = \frac{C_{\text{outSIMO}}}{C_{\text{outSISO}}}$, which is the ratio dividing the two outage capacities, the SISO capacity being computed for a given receive antenna chosen as a reference. In the following, the reference antenna for SISO capacity is antenna with index 6 which is a passive vertical dipole with a 12 m length.

5. Ranking the Different Antenna Configurations

It appears that the outage capacity gains of different efficient antenna configurations are very close from each other. For example, in a simulation of SIMO 1×2 solutions considering 38 antenna combinations and involving 687 valid trials of the channel with variations of month, hour, and azimuth, the 10 largest values of gain are as follows:

2.33, 2.36, 2.41, 2.49, 2.53, 2.68, 3.06, 3.07, 3.18, and 3.32

The four best performances appear as almost equivalent. The reason is the integration in the computation of all possible azimuths ($0-360^\circ$ with a step of 15°) that results in an averaging of the antenna directional responses. Consequently, the ongoing selection does not operate relatively to the only maximal gain: any antenna configuration indicating a capacity gain which exceeds a given proportion of the maximum gain (threshold of 80% in most cases) is regarded as a good candidate. Then, simulations are reiterated with variations of parameters, year, distance, and carrier frequency, with a maximum of 3 (years) \times 5 (distances) \times 5 (frequencies), that is to say, 75 times. Each time, the antenna configurations with a gain exceeding the threshold are identified. For a given antenna configuration, the final ranking is based on the number of occurrences the threshold is exceeded.

6. Set of Antennas Under Test

In this project, a group of 15 possible receive antennas have been considered with a simple geometry due to set up constraints: mobile stations or onboard installation. Complex structures like log periodic or log spiral antennas have been ignored. Antennas with indexes #1 to #5, #12, and #13 are small size active antennas (and additionally the combinations with indexes #14 and #15). Other antennas are passive with a larger size. The list stands in Table 1:

Antenna #6 (vertical dipole) is considered as a reference in the SISO case. Active antennas #1 to #5 are aerials of a device consisting of collocated sensors developed at the Institute of Electronics and Telecommunications of Rennes (IETR) (Figure 1). This device is contained within a 1.7 m side cube and the antenna feed points are 3 m above the ground. Antennas 14 and 15 are combinations of loops #1 and #2. The passive dipoles #8 to #11 are supposed to be set up on a mast 12 m above the ground. In the

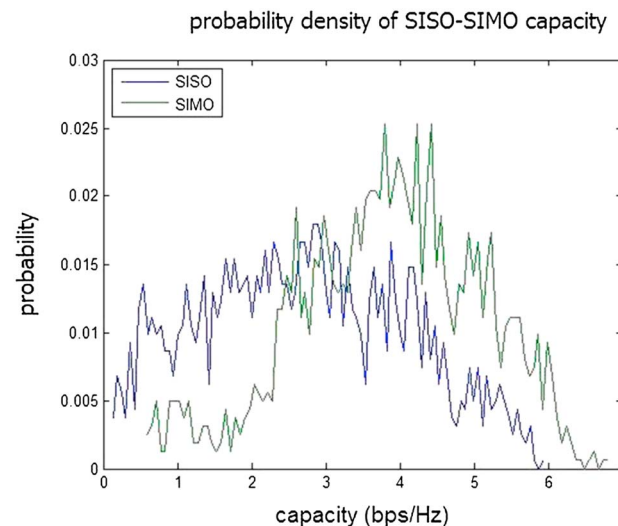


Figure 2. Histograms of capacity.

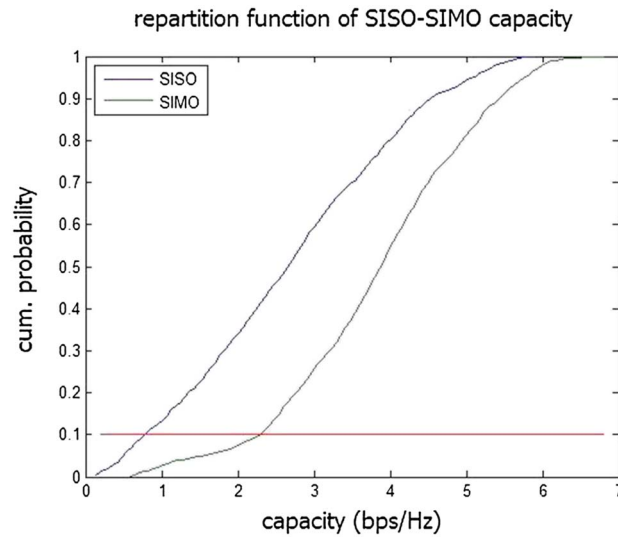


Figure 3. Cumulative histograms of capacity.

have been considered for SIMO 1×2 . The selection procedure starts by computing histograms of capacity for each antenna couple with a large number of channel realizations. For the pair consisting of antennas #1 and #2, an example of histogram based on a set of 1238 realizations obtained for a 900 km link, 9 MHz carrier frequency in year 1969 is plotted in Figure 2. It presents two similar distributions of capacities, but with a shift of the SIMO curve toward higher values of capacities as expected.

The outage capacities are estimated from the cumulated histograms as indicated in Figure 3. With a probability threshold of 0.1, the outage capacity for the SISO case is equal to 0.71 bps/Hz and to 2.20 bps/Hz in the SIMO configuration, which gives a capacity gain of 3.1.

For the same channel realizations, the computation is reiterated for every antenna couple in the set of 38 elements. Figure 4 shows the outage capacity gain as a function of the index of antenna pairs (SIMO 1×2).

In this case, selecting the only couple that reaches the maximum value of gain should be very restrictive as it may not be the best if one of the parameters, distance, frequency, or year, is modified. Therefore, any couple with a gain exceeding 80% of the maximum value will be selected for further evaluation.

Figure 5 indicates, for each couple with index varying from 1 to 38, the number of occurrences of a good ranking (capacity gain exceeding 80% of the maximum value), the total number of trials being equal to 75.

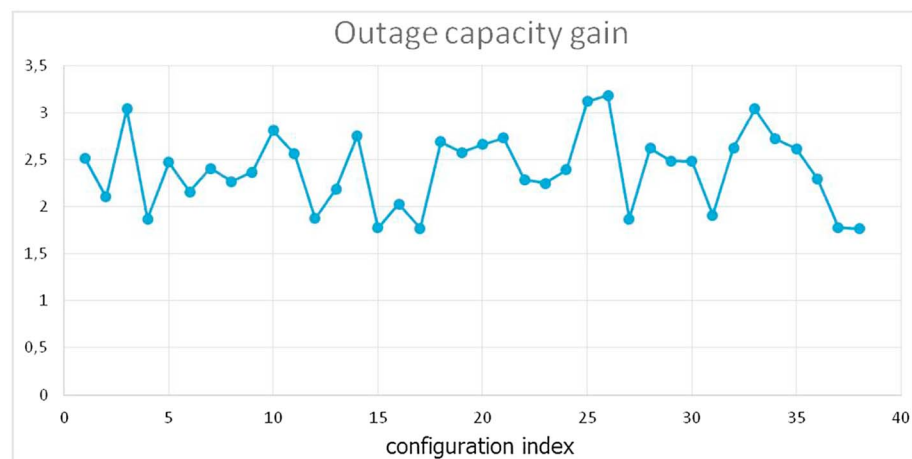


Figure 4. Example of outage capacity gain (38 pairs).

computation of antenna directional responses, the ground effect is taken into account (according to the Sommerfeld method), assuming standard characteristics: a conductivity equal to 0.005 S/m and a permittivity equal to 13. The influence of these ground parameters on the final results has not been considered in this study.

7. Results

In the project, gains in performances (relatively to the SISO case) have been estimated for SIMO systems involving two to five receive channels. In this paper, we limit the presentation to results regarding the optimization of a SIMO 1×2 structure. In the set of 15 receive antennas, 38 pairs

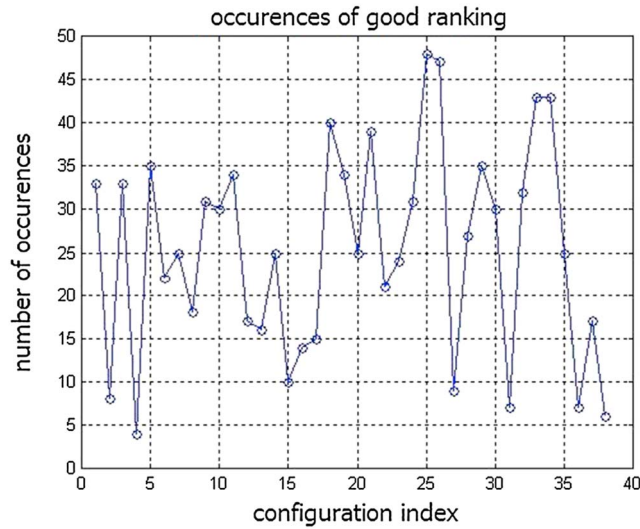


Figure 5. Number of occurrences (trials with a “good” ranking) for 38 pairs of antennas.

It appears that, according to the criterion of a maximal number of occurrences, the best configurations have the indexes 25 and 26 with very comparable performances: they correspond to the association of antennas #12 and #13 for the first one (two orthogonal horizontal dipoles) and antennas #1 and #2 for the second one (two vertical orthogonal loop antennas). The number of iterations obtained by variations of the parameters, year, distance, and frequency, is equal to 50: the maximum number is 75, but all configurations are not valid due to poor propagation previsions or inappropriate shape of the Shannon capacity histogram. Consequently, the actual maximum number of occurrences is 50 and the value reached by the best antenna configuration is equal to 48 (see Figure 5).

With this selection, the typical value of outage capacity gain is close to 3.1. Given the value of the SISO outage capacity (0.72 bps/Hz), the optimal SIMO outage capacity is equal to 2.23 bps/Hz. It can be surprising that the gain value exceeds the number of receive channels (two). But, we must keep in mind that the capacity gain is estimated relatively to a SISO channel including a reference antenna (vertical dipole #6) which may not be the most efficient. With a reference antenna being more effective (as one antenna of couple 25), the capacity gain would be inferior. Moreover, the configuration with index 38 appears as one of the worst choices. It corresponds to the (theoretical) association of two identical vertical dipoles. In that situation, the benefit of a SIMO architecture is an improvement in the SNR (signal-to-noise ratio) but no diversity gain can be expected.

8. Conclusion

This paper proposes a selection criterion for receive antennas set up in a SIMO system for transhorizon radio communication. Subject to available space constraints, the antenna array must present a reduced aperture and consequently involves nonidentical sensors with different directional responses set up with (theoretically) the same phase center. The proposed criterion is based on the outage capacity of SIMO channels, the computation of which resorts to the estimation of channel impulse responses including the receive antenna gains. As the outage capacity is estimated through statistics of Shannon capacity, a large number of trials of ionospheric radio circuits must be considered by variations of year, month, hour, distance, azimuth, and frequency. For each trial, the propagation parameters are predicted by means of the VOACAP software, and the antenna directional responses to the incoming waves are computed with NEC 2-D software.

In the case of SIMO 1 × 2 architecture, two equivalent optimal solutions are identified with the associations of two vertical orthogonal loop antennas or two horizontal orthogonal dipoles. The outage capacity gain (relatively to a SISO solution involving a vertical dipole at the receive end) is close to 3.1, and the corresponding outage capacity is equal to 2.23 bps/Hz. The two antennas of the first couple are elements of the original device designed and built up at the IETR laboratory. Further investigations indicate that an increase in the number of antennas results in an increase in the capacity gain, but with moderate relative variations: maximum capacity gain equal to 3.82 for NC = 3, and 4.31 for NC = 4.

Future work will compare the degrees of diversity provided by the selected SIMO structure with its extension to a MIMO system that we previously proposed in *Ndao et al.* [2013]. In particular, the possibility of an increased throughput under the constraint of equal transmit power needs to be investigated.

Acknowledgments

The data used for this work can be obtained on request from Y. Erhel yvon.erhel@st-cyr.terre-net.defense.gouv.fr.

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