

Comment on "The Sagnac effect and pure geometry" [Am. J. Phys. 83, 427–432 (2015)]

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Comment: "*The Sagnac effect and pure geometry*" [AJP **83**, 427 (2015)]

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Recently, an enlightening article was published in this Journal [1], on the necessary conditions for the observation of the Sagnac effect. The key element, introduced in [1] and not present in several previous articles on the same subject [2,3,4,5,6], is that accelerated motion of the emitted/receiver [7] is not required in order to detect the time delay: even inertial motion is sufficient !

The authors of [1] suggest that their approach can be useful in teaching the foundations of relativity, because the involved calculations are simple. I agree with them and the aim of the present Note is to highlight a special one-dimensional (1D) limit of their example (Section IV of [1]) that, properly complemented with the original discussion [1], should allow for further physical clarifications, as detailed below. Introductory courses usually deal with special relativity in just one spatial dimension so that the main message of [1] might reach in this way an even wider audience.

Consider the light paths of Fig. 1 and Fig. 2, corresponding to those of Section IV of [1], in the limit $b \to 0$. In the reference frame of the physical apparatus supporting the closed light path (the black triangular mirrors), the emitter/receiver (black diamond at the origin) moves right and the light follows two opposite paths: in Fig. 1, light first moves right (thin straight line), is reflected in (a), moves left (dash-dotted line) towards (-a), is reflected again and finally moves right (dashed line) towards the receiver (grey diamond) that has in the meantime moved to $(4a\beta/(1-\beta))$; in Fig. 2 light first moves left (thin straight line), is reflected in (-a), moves right (dash-dotted line) towards (a), is reflected again and finally moves left (dashed line) towards the receiver (grey diamond) that has in the meantime moved to $(4a\beta/(1+\beta))$. The lengths of the two paths are different: therefore the two signals reach the receiver at two different times, $t_{arr1}=4a/(c(1-\beta))$ and $t_{arr2}= 4a/(c(1+\beta))$. The time delay is $\Delta\tau_{mf} = 8\gamma^2 a\beta/c$.

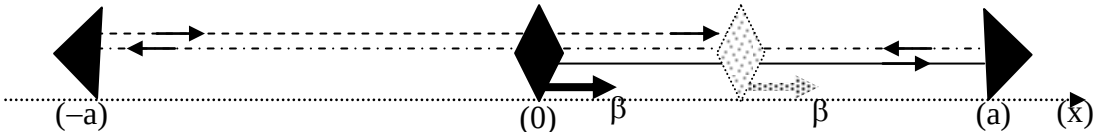


Fig. 1 *– Mirrors reference frame: light is initially emitted at the origin towards right (the black diamond moves right with speed β). The path is longer than in Fig. 2, the reception time as well.*

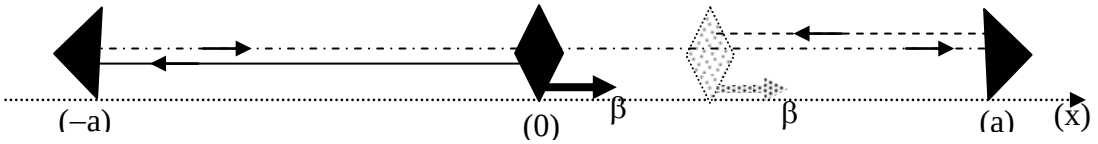


Fig. 2 *– Mirrors reference frame: light is initially emitted at the origin towards left (the black diamond moves right with speed β). The path is shorter than in Fig. 1, the reception time as well.*

In the frame of the emitter/receiver (Fig. 3 and Fig. 4) an analogous simple calculation gives a time delay $\Delta\tau_{rf} = 8\gamma a\beta/c \approx 2\ell v/c$ ($\ell = 4a$). This is of course the same as Eq. (13) of [1] when b=0 (spacetime coordinates of all events can be obtained from [1], with b=0). Two interesting remarks:

1) reception times for Fig. 3 and Fig. 4 can be written, respectively, as $t_{arr3}= 4a\gamma(1+\beta)/c=4a/c\sqrt{((1+\beta)/(1-\beta))}$ and $t_{arr4}= 4a\gamma(1-\beta)/c=4a/c\sqrt{((1-\beta)/(1+\beta))}$, with the same formal dependence in β as the Doppler effect. As it should be, for proper times t_{arr3} and t_{arr4}, we have $t_{arr3}= t_{arr1}/\gamma$ and $t_{arr4}= t_{arr2}/\gamma$.

2) qualitatively, the time delay can be explained in simple terms: the light ray first moving right has to travel a longer path following the left mirror (towards x_2) than the light ray moving first left (towards x_3). In formulae: $x_1 + |x_2| > |x_3| + x_4$.

In our opinion, this 1D example leads to both physical and pedagogical clarifications that usefully complement the two-dimensional (2D) example of [1].

From the physical point of view, two lessons stand clear:

1) The area enclosed by the circuit in one dimension is obviously zero. It is therefore blatant that such an area is not a key element, contrary to what is often assumed since the original Sagnac work [8].

2) The motion being fully 1D, no physical analogy with the Aharonov-Bohm effect is possible, the latter being a topological effect that requires a non-null circulation of the vector potential.

From the pedagogical point of view, the comparison of 1D and 2D examples allows highlighting those elements that, being present in either case, but not in both, are not necessary to the explanation of the Sagnac effect. For example, the clockwise/counter-clockwise paths (in 2D) or left/right paths (in 1D) have no special meaning (see also Fig. 5 below, where a single circuit shows both a clockwise and a counter-clockwise part). Also, the change of shape from rectangular to trapezoidal (Figs. 6 to 9 of [1]), is not a key-ingredient, as the time-delay survives even in the $b \to 0$ limit, where the trapezoid shrinks to a line. Notice, however, that the deformation of an arbitrary closed circuit to the 1D case, as discussed here, is pedagogically clearer when the 2D example of [1] is already known.

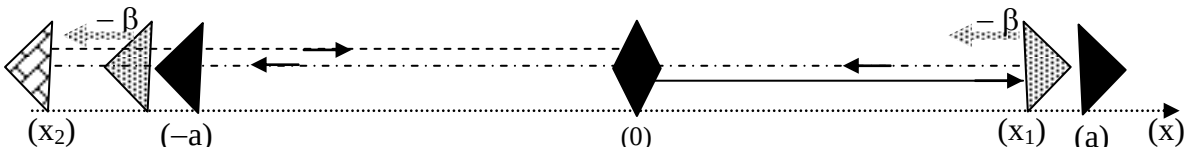


Fig. 3 *– Emitter/receiver reference frame: light is initially emitted by the black diamond fixed at the origin towards right. Mirrors move to left with time-order: black triangle (at t=0), then dotted triangle, then dashed triangle. The path is longer than in Fig. 4, the reception time as well.*

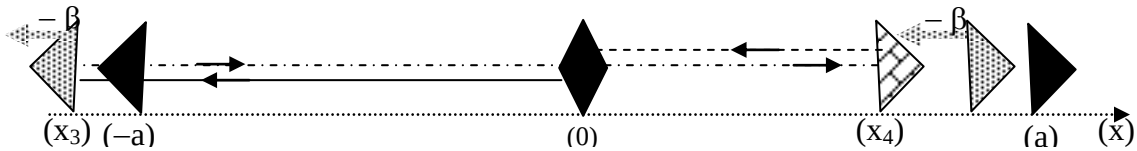


Fig. 4 *– Emitter/receiver reference frame: light is initially emitted by the black diamond fixed at the origin towards left. Mirrors move towards left with time-order: black triangle (at t=0), then dotted triangle, then dashed triangle. The path is shorter than in Fig. 3, the reception time as well.*

To conclude, I believe that the original 2D example of [1] and the present 1D example complement each other and corroborate the main message of [1]: the Sagnac effect is a purely geometrical effect determined by the different lengths of the global light paths in the left/right (or clockwise/counter-clockwise) circuits. Coupled to the constant speed of light in any inertial frame, this necessarily leads to a time delay. As the above 1D discussion clearly shows, the effect does not depend on the area of the circuit and it is not a topological effect. In this sense, it is interesting to stress that also closed paths that are topologically non-equivalent to the cylinder of Fig. 2 of [1] are characterized by a Sagnac effect (like in Fig. 5, $\Delta\tau = 48a\gamma^2\beta/c \approx 2\ell v/c$ – here $\ell = 24a$, we leave it as an exercise for the interested reader). But this is another story...

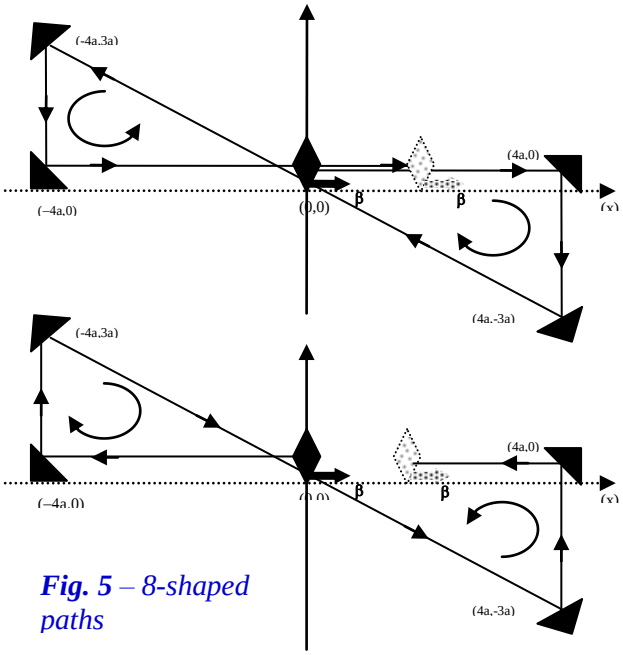


Fig. 5 *– 8-shaped paths*

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[7] To be precise, we should refer to the motion of the emitter/receiver with respect to the physical apparatus supporting the closed light path. Two light rays are emitted in opposite directions in it.
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