Hardware-friendly DST-VII/DCT-VIII approximations for the Versatile Video Coding Standard

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***Abstract*—Versatile Video Coding (VVC) is the next generation video coding standard expected by the end of 2020. The new concept of Multiple-Transform Selection (MTS) has been introduced in VVC. MTS enables the VVC encoder to select the transform that minimizes the rate-distortion cost among a set of pre-defined trigonometric transforms including the well known Discrete Cosine Transform (DCT)-II, DCT-VIII and Discrete Sine Transform (DST)-VII. Unlike the DCT-II that has fast computing algorithms, the DST-VII and DCT-VIII rely on more complex matrix multiplication.**

This paper tackles the problem of DST-VII and DCT-VIII approximations based on the DCT-II and an adjustment stage. This latter consists in a multiplication by a band-matrix with low number of non-zero coefficients per row. The approximation problem is first modeled as a constrained integer optimization problem minimizing both error and orthogonality. The genetic algorithm is then used to solve the optimization problem and find the adjustment band-matrix that minimizes a trade-off between error and orthogonality. The proposed solution enables to preserve the coding gain achieved by the MTS and considerably reduces the complexity in terms of required number of multiplications by coefficient. Moreover, the proposed approach is hardware-friendly and will provide a lightweight shared hardware module for DST-II, DST-VII and DCT-VIII transforms.

***Index Terms*—Multiple-Transform Selection, DCT, DST, Approximation, VVC.**

I. Introduction

The next generation video coding standard named Versatile Video Coding (VVC) is under development by the Joint Video Experts Team (JVET), established by Motion Picture Experts Group (MPEG) and Video Coding Experts Group (VCEG) [1]. The VVC standard, expected by the end of 2020, introduces several new coding tools enabling around 30% [2] of coding gain beyond High Efficiency Video Coding (HEVC) standard. This coding gain is mainly enabled through enhancements of encoder blocks including frame partitioning, intra/inter predictions, transform and in-loop filters. A new concept of transform called Multiple-Transform Selection (MTS) has been introduced involving several Discrete Cosine Transform (DCT)/Discrete Sine Transform (DST) cores. Besides the usual DCT-II used in video coding standards, the encoder selects combinations of DCT-VIII and DST-VII, for the horizontal and vertical transforms, to optimize the Rate Distortion (RD) cost J, a trade-off between distortion D and rate R [3]

$$J = D + \lambda R. \tag{1}$$

This solution brings a significant coding gain ($\sim 2\%$ to 0.9% bit-rate reduction in VTM-4.0 [4]) compared to HEVC which considers only DCT-II along with DST-VII for Intra luma blocks of size 4×4 [5]. However, this coding gain comes at the expense of significant increase in both codec memory usage and its complexity, estimated to 160% encoder and 105% decoder complexities increase in All Intra (AI) configuration [4]. While the DCT-II has been well studied and optimized with fast implementations [6]–[8], the DST-VII/DCT-VIII do not have efficient fast implementation algorithms [9], [10].

To provide a fast implementation for the DST-VII and DCT-VIII, this paper proposes to approximate those basis with a combination of the DCT-II basis and a sparse adjustment matrix A. In that way, existing DCT-II architecture can be reused and the DST-VII computation impact is reduced. The integration of this solution in the VVC standard would significantly reduce the computational cost of the transform module at both encoder and decoder especially for hardware implementation on embedded platforms with limited computing and memory resources.

The rest of this paper is organized as follows. Section III gives the theoretical derivation of the DST-VII approximation expressed as a constrained discrete optimization problem. The genetic based algorithm is presented in Section IV to solve the optimization problem and compute the coefficients of the sparse band matrix A. Section V gives the performance of the approximate DST-VII and DCT-VIII in terms of both coding efficiency and complexity. Finally, Section VI concludes the paper.

II. Related work and motivations

In this paper we focus on the approximation of the DST-VII transform. The DCT-VIII can then be derived from the DST-VII at no additional computational complexity, involving only vector reflection matrix Γ and sign changes matrix Λ as expressed in Equation (2)

$$C_8 = \Lambda \cdot S_7 \cdot \Gamma, \tag{2}$$

where C_8 and S_7 are the coefficients matrices of DCT-VIII and DST-VII transforms, respectively, while matrices Λ and

TABLE I
PERFORMANCE IN TERMS OF NUMBER OF MULTIPLICATIONS AND ADDITIONS OF THE FAST COMPUTING ALGORITHMS OF DCT-II AND DST-VII

Transforms	$N=8$			$N=16$			$N=32$			$N=64$		
	Ref.	+	×	Ref.	+	×	Ref.	+	×	Ref.	+	×
DCT-II	[7]	29	11	[7]	81	31	[8]	209	80	[8]	192	513
DCT-II (HEVC)	[5]	37	24	[5]	113	86	[5]	401	342	[5]	807	683
DST-VII	[9]	77	21	[10]	150	146	–	–	–	–	–	–
DST-VII	[11]	–	–	[11]	155	127	[11]	718	620	[11]	2331	2207
Matrix Multip.	–	56	64	–	240	256	–	992	1024	–	4032	4096
Prop. DST-VII ($\theta=4$)	–	61	56 (88%)	–	161	150 (58%)	–	497	470 (45%)	–	999	939 (23%)
Prop. DST-VII ($\theta=5$)	–	69	64 (100%)	–	177	166 (64%)	–	529	502 (49%)	–	1063	1003 (24%)
Prop. DST-VII ($\theta=6$)	–	77	72 (113%)	–	193	182 (71%)	–	561	534 (52%)	–	1127	1067 (26%)

Γ are computed by Equations (3) and (4), respectively.

$$\Gamma_{i,j} = \begin{cases} 1, & \text{if } j = N-i+1, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

$$\Lambda_{i,j} = \begin{cases} (-1)^{i-1}, & \text{if } j = i, \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

with $i,j \in \{1,2,\dots,N\}$ and N is the transform size. Several studies have investigated fast computing algorithms for DCTs/DSTs. They aim at reducing the required number of multiplications and additions compared to the matrix multiplication requiring for $N{\times}N$ square matrix N^3 multiplications and $N^2(N-1)$ additions (ie. $\mathcal{O}(N^3)$ computational complexity). This computational complexity can be further reduced to $\mathcal{O}(N^{2.373})$ which is the state of the art lower bound [12]. Some of the DCTs/DSTs offer decomposition, symmetry and recursion properties enabling to design fast and low complexity algorithms. The 1D transform of a residual vector x of size $N{\times}1$ using the DST-VII transform S_7 is expressed as follows

$$y = S_7 \cdot x. \tag{5}$$

Table I gives the computational complexity in terms of number of multiplications and additions of the existing fast implementations for both DCT-II and DST-VII 1D transforms to process a vector x of size $N \in \{8, 16, 32, 64\}$. We can notice that the number of operations remains low compared to the naive multiplication for DCT-II while it significantly increases for DST-VII especially for N larger than 16. Authors in [11] have explored three properties of the DST-VII basis that enable factorisation for fast implementation algorithm. These properties enable to gather several multiplications by coefficients of the same absolute value within a basis row in one multiplication operation. This simple factorisation technique enables to reduce the number of operations required by matrix multiplication without introducing any approximation or coding loss. Reznik has shown in [13] the existing connection between DCT-II and DST-VII transforms. This relationship enables their joint computation for certain transform sizes. It has also been shown in [14] that DCT-II of odd size is equivalent to computing the same length Discrete Fourier Transform (DFT). Thus, the fast DFT algorithm can be used to compute certain sizes of DCT-II and DST-VII transforms.

The objective of this work is to propose approximations for DST-VII at large sizes (ie. $N \in \{16, 32, 64\}$) that meet the following three main requirements:

- Lightweight solution with lower number of operations than the existing fast computing algorithms proposed in [10], [11] for large transform sizes and lower memory usage to store the transform coefficients.
- Hardware-friendly: use the existing forward DCT-II and inverse DCT-II fast implementations to preserve memory and logic resources of hardware platforms.
- Preserve the coding efficiency achieved by the MTS under the VVC Common Test Conditions (CTC) [15].

III. PROBLEM FORMULATION

In this section, the approximation of the DST-VII is expressed as a constrained optimization problem. The approximate DST-VII is expressed according to the DCT-II in Equation (6) [16], [17]

$$\hat{S}_7 = \Lambda \cdot C_2^T \cdot \Gamma \cdot A, \tag{6}$$

where $\Lambda \cdot C_2^T \cdot \Gamma$ is equivalent to the DST-III transform, Λ and Γ matrices are computed by Equations (3) and (4), respectively and A being a sparse band matrix. The inverse approximate DST-VII is used at the decoder side involving the use of forward DCT-II

$$\hat{S}_7^T = A^T \cdot \Gamma \cdot C_2 \cdot \Lambda. \tag{7}$$

This approximate DST-VII $\hat{S}_7$ and its transpose, initially proposed in [16], enable to reduce its computational complexity since it involves both the DCT-II that has fast computing algorithms (Table I) and a multiplication by a band matrix A. Therefore, the complexity of this approximation in number of operations is equal to the complexity of the DCT-II plus the complexity related to multiplication by the band matrix A which depends on the maximum number of non-zero coefficients by row θ. The complexity of the multiplication by the matrix A in terms of numbers of multiplications and additions are given by $\theta\, N$ and $(\theta-1)\, N$, respectively. In this paper three values of non-zero coefficients by row are considered $\theta \in \{4, 5, 6\}$. Table I shows that the three configurations of the approximate DST-VII require lower number of operations in terms of both additions and multiplications compared to the existing DST-VII fast implementations for large transform size (ie. $N > 16$). These numbers include the complexity related to multiplication by the sparse matrix A plus the complexity of the fast DCT-II (HEVC) transform [5]. Moreover, this architecture is suitable for hardware implementation since the logic and memory resources of DCT-II transform would also be used to process DST-VII and DCT-VIII transforms. The proposed approach consists in minimizing the weighted least-squares error between the DST-VII S_7 and its approximated version $\hat{S}_7$

$$E(A) = \sum_{i=1}^{N} \omega_i \sum_{j=1}^{N} \left(S_{7\,i,j} - \hat{S}_{7\,i,j} \right)^2, \tag{8}$$

where ω_i, $i \in \{1, \ldots, N\}$ is a weight vector of size N which might account for the relative importance of the frequency components. When the ω_i is constant equal to 1, the error function corresponds to the squared Frobenius norm.

The approximate DST-VII transform $\hat{S}_7$ shall meet two constrains related to the matrix A that needs to be sparse and orthogonal. This latter is an important property of the transform core since it enables the use of its transpose in inverse transform to recover the original signal. The orthogonality of the adjustment matrix A can be expressed by Equation (9)

$$O(A) = ||A \cdot A^T - I||_2^2, \tag{9}$$

where I is the identity matrix and $|| \cdot ||_2$ stands for the Euclidean norm. Therefore, the objective function of this constrained optimization problem can be expressed with a Lagrangian multiplier λ as follows

$$\underset{A}{\text{minimize}} \quad E(A) + \lambda\, O(A). \tag{10}$$

This objective function aims at minimizing the trade-off between error $E(A)$ and orthogonality $O(A)$ of the approximate DST-VII $\hat{S}_7$ where this trade-off is tuned by the Lagrangian parameter λ. The optimal solution of the optimization problem of Equation (10) consists in the matrix $A^\star$ that leads to the the original DST-VII S_7 expressed as follows

$$A^\star = \Gamma \cdot C_2 \cdot \Lambda \cdot S_7, \tag{11}$$

with $E(A^\star)$ and $O(A^\star)$ terms are both equal to zero. However, this optimal solution is not appropriate as it does not provide integer values, as required for video codecs, and does not reveal a sparse property, leading to fewer arithmetic operations. Let us consider the $A^\star$ matrix with values multiplied by 2^β (with β the bit-depth set to 7 bits) and rounded to the nearest integer. This matrix has its most significant absolute values around the diagonal and lower absolute values are located at lower-left and upper-right parts of the matrix. This property of the adjustment matrix A is stronger for adjustment matrices of higher sizes $N \in \{16, 32, 64\}$.

In this paper, adjustment band matrix that minimizes the trade-off between error and orthogonality is sought with the constraint of A to include few integer values different from zero. This discrete constrained optimization problem is expressed as follows

$$\begin{aligned} \underset{A}{\text{minimize}} \quad & E(A) + \lambda\, O(A) \\ \text{subject to} \quad & A_{i,j} = 0, \quad \forall\, j > i + \lfloor \theta/2 \rfloor \\ & A_{i,j} = 0, \quad \forall\, j \le i - \lceil \theta/2 \rceil, \\ & i, j \in \{1, \ldots, N\}^2, \\ & A_{i,j} \in \mathbb{Z} \cap [\, -2^\beta + 1,\, 2^\beta\,], \\ & \lambda \in \mathbb{R}^+ \end{aligned} \tag{12}$$

It has been shown in [18] that the DST-VII is optimal in terms of energy packing for image intra-predicted residuals. Indeed, those residuals have an auto-correlation matrix which is a tri-diagonal matrix R_x of size $N{\times}N$ expressed by Equation (13)

$$R_{x\ i,i} = b,\ R_{x\ i,i+1} = c,\ R_{x\ j-1,j} = a,\ R_{x\ N,N} = b - \alpha, \tag{13}$$

with $(a, b, c, \alpha) = (-1, 2, -1, 1)$ and $1 \le i < N\,,\ 1 < j \le N$. The eigen-vectors of the matrix R_x are the basis of the DST-VII transform [19]. Therefore, for the approximation of the DST-VII, we propose to weight the relative importance of the approximation basis with the eigenvalues of the matrix R_x. This gives more importance to the lower frequency range where an important part of the signal energy stands. According to [20] the eigenvalues are

$$\omega_i = 2 \left(1 + \cos\left(\frac{2\, i\, \pi}{2\, N + 1} \right) \right), \quad i = 1, \ldots, N. \tag{14}$$

IV. Genetic search algorithm

To provide an approximation of the DST-VII, the adjustment matrix, which consists of a selected number θ of integer values around the diagonal, needs to be determined for a desired level of orthogonality $O(A)$ expressed in Equation (10).

To solve this problem in integer domain, continuous optimization methods such as gradient descent are not appropriate. Also, an exhaustive search would result in evaluating large number of combinations $\left(2^{\beta+1} + 1\right)^{\theta N}$. Techniques such Integer Programming [21] can provide helpful tools in that context, however in this study a genetic algorithm approach was preferred as it provided satisfactory results and appeared to converge well.

Genetic algorithm, are easily re-configurable to address varying scenarios such that the adjustment matrix with different number of coefficients per row. Indeed, this optimization algorithm solves the problem (10) with $\theta\, N$ parameters with the same strategy. Basically, it consists in changing individual elements of the adjustment matrix in the *mutation* process.

Although convergence is not guaranteed with the Genetic Algorithm approach, it appears in practice that it converges in a consistent fashion with different initialization points.

The principle of the genetic search is the following:

- From a set of N_p selected adjustment matrices, called parents, N_c children are created by individual changes in the close-to-diagonal values. One among the children's values, randomly selected, is changed by the addition of +/-1 while ensuring that the value remain in the adjustment matrix bit-depth range.

- The resulting $N_p N_c$ adjustment candidate matrices are evaluated by Equation (10), this can be done in parallel, e.g. using OpenMP programming interface [22].
- From the candidate matrices, $N_p - 1$ are randomly retained, and the best performing matrix is kept. From these N_p matrices the three steps are re-iterated until convergence of the algorithm.

As the A matrices have coefficients around the diagonal, the number of parameters is function of the matrix size and the number of coefficients per row θ. It is in the range of $\theta\, N$, each coefficient is to be expressed on β bits for implementation on fixed-point devices. The convergence is measured in terms of stabilization of the algorithm, that is no further reduction of the optimized metric after many iterations.

The λ value is modified in order to provide variation in the approximation / orthogonality space. It is essential in video coding to provide transforms with a reconstruction level sufficiently low to avoid the introduction of distortion in the transform process. To avoid this, λ needs to be chosen such that the orthogonality measure $O(A)$ is in the range of -60 dB.

Fig. 1 gives the performance of the approximate DST-VII for different λ values, the three considered θ values and transform size $N = 32$. The different points of each curve correspond to different values of λ.

The red curve corresponds to $\theta = 5$ with an additional constraint of symmetry across the diagonal between non-zero coefficients

$$\begin{aligned} A(j,i) &= -A(i,j),\ \forall\, j = i+1,\ \ i \in \{1,2,\ldots,N-1\},\\ A(j,i) &= A(i,j),\ \forall\, j = i+2,\ \ i \in \{1,2,\ldots,N-2\}. \end{aligned} \tag{15}$$

This enables a reduced storage of the adjustment matrix with faster convergence of the Genetic algorithm since less possible configurations are assessed with the additional constrains in the order of $N + (\theta - 1)/2$.

The accuracy of the approximate DST-VII enhances in both proximity and orthogonality with the increase of non-zero coefficients in the matrix A. The particular symmetry property (red curve) enables to save the memory required to store the adjustment matrix coefficients. Moreover, the specific ordering of the coefficients is more convenient for Single Instruction Multiple Data (SIMD) and hardware implementations [23]. However, this additional constraint decreases the performance of the approximate DST-VII in terms of both orthogonality and error with respect to $\theta = 5$ configuration, illustrated by the blue curve.

V. Experimental results

A. Experimental Setup

The coding and complexity performance of the black point $N = 32$ on the red curve in Fig. 1 (ie. $\theta = 5$ + Symmetry) together with two other points of sizes $N \in \{16, 64\}$ are investigated in this section under the VVC CTC [15]. Those

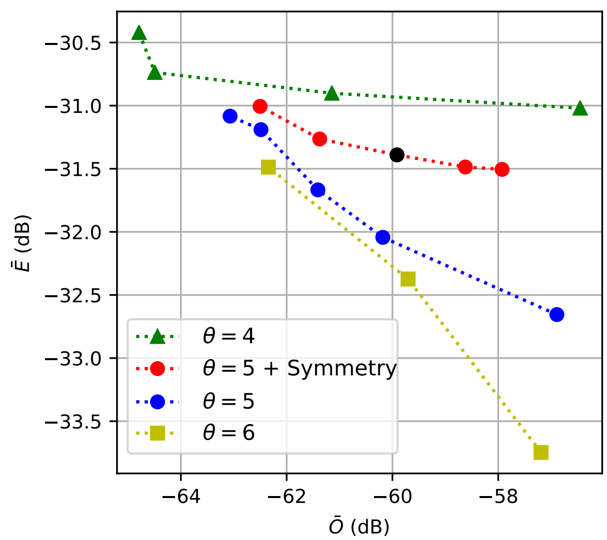

Fig. 1. Performance of approximate DST-VII transform $N = 32$.

experiments are tested among mandatory video classes, where each class corresponds to a specific resolution (up to 4K video) and video content characteristic (with computer generated and visio-conference materials). The proposed approximate DST-VII and DCT-VIII have been integrated in the VVC Test Model (VTM) reference software draft 3.0 [24] (Anchor). The BD-BR metric is used to assess the coding performance over four bitrates between two coding configurations giving the bitrate gain/loss $(-/+)$ in percentage for similar Peak Signal to Noise Ratio (PSNR) quality. The encoding $EncT$ and decoding $DecT$ run times are also compared in percentage to the anchor VTM3.0.

B. Results

Table II gives the performance is terms of both BD-BR and complexity of the proposed solution with respect to the anchor (VTM3.0) codec. This latter uses the HEVC DCT-II together with DST-VII and DCT-VIII core transforms for MTS, up to size 32, implemented as matrix multiplications. It should be noted that disabling the MTS in the VTM software introduces in average 1.95% BD-BR loss with 63% and 95% encoder and decoder complexity reductions respectively, in AI configuration.

From Table II it is shown that the proposed approximations of the DST-VII and DCT-VIII introduce limited coding loss of 0.09% in average for the luminance component (Y); 0.01% and 0.02% for the two chrominance components (U and V) in AI coding configuration. Small gains are also observed for the Class A1 sequences, as 64-sized MTS is enabled with the proposed approach. Overall, we can conclude that the coding performance remains similar to the anchor for Random Access (RA) and Low Delay B (LDB) Inter coding configurations.

The encoding and decoding run times slightly decrease with the approximate DST-VII and DCT-VIII in AI configuration, while they remain constant in RA and LDB configurations. In fact, the gain in number of multiplications and additions

TABLE II
PERFORMANCE (%) IN TERMS OF BJØNTEGAARD DELTA RATE (BD-BR) AND RUN TIME COMPLEXITY OF APPROXIMATE DST-VII AND DCT-VIII

Class	All Intra Main 10					Random Access Main 10					Low Delay B Main 10				
	Y	U	V	*EncT*	*DecT*	Y	U	V	*EncT*	*DecT*	Y	U	V	*EncT*	*DecT*
A1	-0.01	-0.18	-0.08	96	80	-0.15	-0.48	-0.46	102	97	–	–	–	–	–
A2	0.14	0.10	0.03	96	84	0.03	0.08	0.06	101	98	–	–	–	–	–
B	0.12	0.06	0.08	96	83	0.05	0.00	-0.17	101	98	0.04	-0.35	-0.10	101	101
C	0.07	-0.02	-0.06	97	89	0.05	0.05	0.24	100	100	0.05	0.36	0.10	101	102
E	0.13	0.04	0.13	96	86	–	–	–	–	–	0.15	0.26	0.24	100	97
Av.	0.09	0.01	0.02	96	85	0.01	-0.07	-0.07	101	98	0.07	0.04	0.05	101	100
F	0.07	0.16	0.14	96	92	0.05	0.22	0.28	100	98	0.12	0.46	0.34	101	102

enabled by the approximate transforms through adjustment matrices is low in the context of the VTM software, which includes other time consuming operations. However, this gain in number of operations as well as in memory usage has a significant impact in the context of hardware implementation on Field-Programmable Gate Array (FPGA) and ASIC platforms with limited logic and memory resources [23], [25].

VI. CONCLUSION

In this paper, approximations of the DST-VII/DCT-VIII are modeled as a constrained integer optimization problem minimizing the trade-off between error and orthogonality. The genetic algorithm is then used to solve the problem and find the optimal adjustment band-matrix that minimizes the error for a certain level of orthogonality.

The performance of the proposed solution has been assessed in the context of the VVC reference software. The approximate transforms allow to preserve the coding gain enabled by the MTS in both Intra and Inter coding configurations. Moreover, the proposed solution is hardware-friendly since it decreases both the number of required multiplications by 25% for $N = 64$ and memory required to store the transform coefficients.

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